ceived a peach? Mary had three apples, she gave two to her sister and one to her cousin, how many had she left? How many is two times one? Three times one? How often is one contained in two? in three? Jane has three cents, and wishes to buy pens, which cost two cents each, how many can she buy? how much will she have left? &c., &c.

The teacher similarly proceeds with the analysis of the number FOUR. *One* ball and *one* ball and *one* ball and *one* ball are *four* balls; four balls are four times *one* ball; *two* balls and *two* balls are four balls; four balls are two times *two* balls.

(To be continued in next No.)

HOW TO TEACH MENSURATION.

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II.

TRIANGLE.

CASE I. To find the area of a triangle when the base and altitude are given.

1. Begin the lesson by giving three or four simple examples in finding the area of a rectangle.

Example. Find the area of a rectangle whose base is 24 and altitude 14.

2. Cut, out of paper, a rectangle whose base and altitude are, say 12 and 8 respectively. Pin it on the blackboard, and have the class find its area.

3. While the rectangle is pinned on the board, cut out of it a triangle whose base is the base of the rectangle, and altitude the altitude of the rectangle. Then lay the pieces which are cut off upon the triangle.

Question the class in the following manner:

Ques. How does the size or area of the pieces compare with the size or area of the triangle?

Ans. The area of the pieces is equal to the area of the triangle.

Ques. How does the area of the triangle compare with the area of the rectangle?

Ans. The area of the triangle is one half the area of the rectangle.

Ques. If you had the area of the rectangle, how would you find the area of the triangle?

Ans. I would take half the area of the rectangle to find the area of the triangle.

Ques. If you had the base of a triangle and the altitude, how would you find the area?

Ans. and Rule. I would multiply the base by the altitude, which would give the area of the rectangle, and then take half of it for the area of the triangle.

Ques. How could you find the area without taking half the product of the base into the altitude?

Ans. I would multiply the base by half the altitude, or multiply the altitude by half the base.

Ques. What would be the area of the rectangle that was pinned on the board?

Ans. The area would be 12 multiplied by 8, which is 96.

Ques. What would be the area of the triangle that was cut out of it?

Ans. The area of the triangle would be the base 12, multiplied by 4 (half the perpendicular), which is 48.

Ques. What are the factors of the area of a triangle?

Ans. The factors of the area are the base and half the altitude, or the altitude and half the base.

Ques. If you had the area of a triangle and the base, how would you find the altitude?

Ans. I would divide the area by half the base to find the altitude.

1. Find the area of a triangle whose base is 64 and altitude 36.
2. Find the altitude of a triangle whose area is 462 and base 42.
3. Find the base of a triangle whose area is 306 and altitude 18.
4. Find the number of acres in a triangular field whose base is 32 rods and altitude 26 rods.

Geometrical proof, Euclid I. 41.

CASE II. To find the area of a triangle when the three sides are given.

Mechanical proof. None.

RULE.

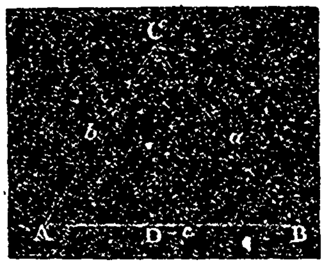
From half the sum of the three sides subtract each side severally; multiply together the half sum and the three remainders, and extract the square root of the product.

DEMONSTRATION.

1. *When the three sides of a triangle are given, and a perpendicular let fall from the vertical angle upon the base, to find the segments of the base.*

Let a, b, c denote the sides of the triangle ABC; then, by Euclid I. 47, $b^2 - AD^2 = CD^2$, for the same reason $a^2 - DB^2 = CD^2$; $\therefore b^2 - AD^2 = a^2 - DB^2$ and $a^2 - b^2 = DB^2 - AD^2$, and $(DB+AD)(DB-AD) = a^2 - b^2$.

$$DB - AD = \frac{a^2 - b^2}{DB + AD} = \frac{a^2 - b^2}{c}, \text{ } c \text{ being} = AD + DB.$$



Therefore $\frac{a^2 - b^2}{c}$ is equal to the difference between the segments DB and AD; and $c - \frac{a^2 - b^2}{c}$ = twice the shorter segment,

$$\therefore \text{shorter segment} = \tfrac{1}{2}\left(c - \frac{a^2 - b^2}{c}\right) = * \frac{c^2 + b^2 - a^2}{2c}.$$

RULE.

To find the segment of the base, from the square of the base and the square of one of the sides subtract the square of the other side, and divide the remainder by twice the base.

2. *To find the perpendicular let fall upon the base.*

Euclid I. 47. $AC^2 - AD^2 = CD^2 \quad \therefore b^2 - * \left(\frac{c^2 + b^2 - a^2}{2c}\right)^2 = \frac{4b^2c^2 - (b^2 + c^2 - a^2)^2}{4c^2} \quad \therefore CD$, the perpendicular $= \frac{\sqrt{4b^2c^2 - (b^2 + c^2 - a^2)^2}}{2c}$.

3. *To find the area.*

$$\text{Area by case I.} = \frac{AB \times CD}{2} = \frac{\sqrt{4b^2c^2 - (b^2 + c^2 - a^2)^2}}{4c} \times c$$
$$= \tfrac{1}{4}\sqrt{4b^2c^2 - (b^2 + c^2 - a^2)^2}.$$

The quantity under the radical sign being the difference between two squares may be resolved into the factors $(2bc + (b^2 + c^2 - a^2)$ $(2bc - (b^2 + c^2 - a^2)$; and these in the same way may be resolved into $(b + c + a)$ $(b + c - a)$ and $(a + b - c)$ $(a - b + c)$.