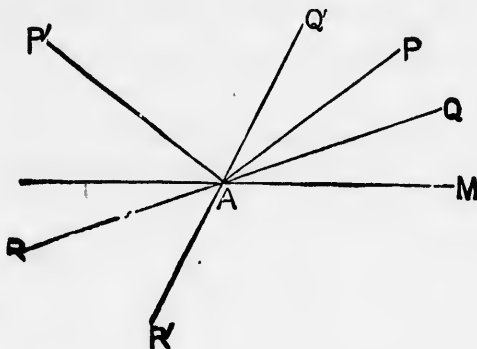


Now the ratios of all these angles are of the same magnitude and can only differ in sign, there being a pair of angles which have each ratio + and a pair—.

$\therefore$  in determining from the cosine we get *two* values.

Next, let  $A$  be an angle whose sine is known, and describe  $MAP$  the least angle which has the given sine.

Make  $M\hat{A}P'$  equal to the supplement of  $M\hat{A}P$ .



Then  $A$  must be either  $M\hat{A}P$  or  $M\hat{A}P'$ , or some angle formed by adding (or taking) a multiple of four right angles to (or from) either of these.

Bisect these angles as before

Then  $\frac{A}{2}$  must be an angle which has  $MA$  and one of the four  $QA$ ,  $RA$ ,  $Q'A$ ,  $R'A$  for its bounding lines.

Now those which have either  $QA$  or  $RA$  as one of their boundaries have ratios equal in magnitude but opposite in sign. So also for those having  $Q'A$  or  $R'A$  as one of their boundaries; but the magnitudes of the ratios of the former sets differ from those of the latter.

$\therefore$  the ratios of  $\frac{A}{2}$  may be either of two sets of magnitudes and of either sign, and therefore have *four* different values.