SOAP-BUBBLES,

AND THE FORCES WHICH MOULD THEM.

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(Continued.)

Let us therefore examine this case more in detail. I have a disc of card which has exactly the same diameter as the waist of the cast. I now hold this edgeways against the waist (Fig, 27), and though you can see that it does not fit the whole curve, it fits the part close to the waist perfectly. This then shows that this part of the cast would appear curved inwards if you looked at it sideways, to the same extent that it would appear curved outwards if you could see it from above. So considering the waist only, it is curved both towards the inside and also away from the inside according to the way you look at it, and to the same extent. The curvature inwards would make the pressure inside less, and the curvature outwards would make it more, and as they are equal they just balance, and there is no pressure at all. If we could in the same way examine the bubble with the waist, we should find that

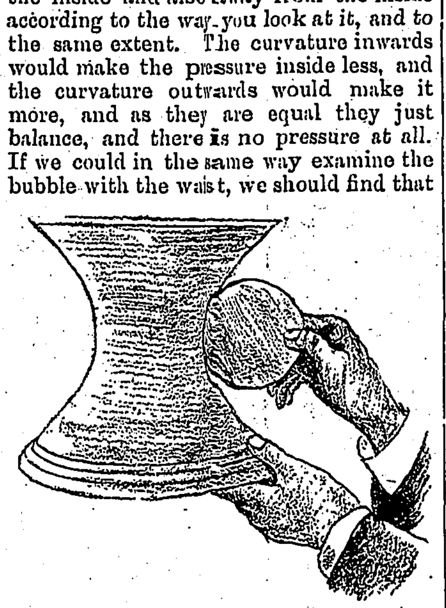

FIG. 27.

this was true not only at the waist but at every part of it. Any curved surface like this which at every point is equally curved opposite ways, is called a surface of no curvature, and so what seemed an absurdity is now explained. Now this surface, which is the only one of the kind symmetrical about an axis, except a flat surface, is called a catenoid, because it is like a chain, as you will see directly, and, as you know, *catena* is the Latin for a chain. I

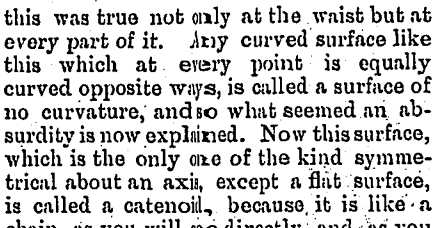

FIG. 28.

shall now hang a chain in a loop from a level stick, and throw a strong light upon it, so that you can see it well (Fig 28). This is exactly the same shape as the side of a bubble drawn out between two rings, and open at the end to the air.

Let us now take two rings, and having placed a bubble between them, gradually alter the pressure. You can tell what the pressure is by looking at the part of the film which covers either ring, which I shall call the cap. This must be part of a sphere, and we know that the curvature of this and the pressure inside rise and fall together. I have added now just the bubble so that it is a nearly perfect sphere. If I blow in more air the caps become more curved, showing an increased pressure, and the sides bulge out even more than those of a sphere (Fig 29). I have now brought the whole bubble back to the spherical form. A little increased pressure, as shown by the increased curvature of the cap, makes the sides bulge more; a little less pressure, as shown by the flattening of the caps, makes the sides bulge less. Now the sides are straight, and the cap, as we have already seen, forms part of a sphere of twice the diameter of the cylinder. I am still further reducing the pressure until the caps are plane, that is, not curved at all. There is now no pressure inside, and therefore the sides have, as we have already seen, taken the form of a hanging chain; and now, finally, the pressure inside, is less than the outside, as you can see by the caps being drawn inwards, and the sides have even a smaller waist than the catenoid. We have now seen seven curves as we gradually reduced the pressure, namely—

1. Outside the sphere.
2. The sphere.
3. Between the sphere and the cylinder.
4. The cylinder.
5. Between the cylinder and the catenoid
6. The catenoid.
7. Inside the catenoid.

Now I am not going to say much more about all these curves, but I must refer to

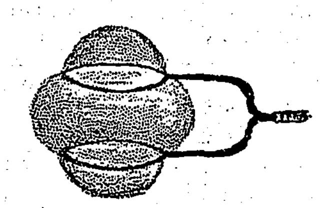

FIG. 29.

the very curious properties that they possess. In the first place, they must all of them have the *same* curvature in every part as the portion of the sphere which forms the cap; in the second place, they must all be the curves of the least possible surface which can enclose the air and join the rings as well. And finally, since they pass insensibly from one to the other as the pressure gradually changes, though they are distinct curves there must be some curious and intimate relation between them. This, though it is a little difficult, I shall explain. I shall show you a simple experiment which will throw some light upon the subject, which you can try for yourselves at home.

I have here a common bedroom candlestick with a flat round base. Hold the candlestick exactly upright near to a white wall, then you will see the shadow of the base on the wall below, and the outline of the shadow is a symmetrical curve, called a hyperbola. Gradually tilt the candle away from the wall, you will then notice the sides of the shadow gradually branch away less and less, and when you have so far tilted the candle away from the wall that the flame is exactly above the edge of the base,—and you will know when this is the case, because the falling grease will just fall on the edge of the candlestick and splash on to the carpet,—I have it so now,—the sides of the shadow near the floor will be almost parallel (Fig 30), and the shape of the shadow will have become a curve, known as a parabola; and now when the candlestick is still more tilted, so that the grease misses the base altogether and falls in a gentle stream upon the carpet, you will see that the sides of the shadow have curled round and met on the walls, and you now have a curve like an oval, except that the two ends are alike, and this is called an ellipse. If you go on tilting the candlestick, then when the candle is just level, and the grease pouring away, the shadow will be almost a circle; it would be an exact circle if the flame did not flare up. *Now if you go on tilting the* candle, until at last the candlestick is upside down, the curves already obtained

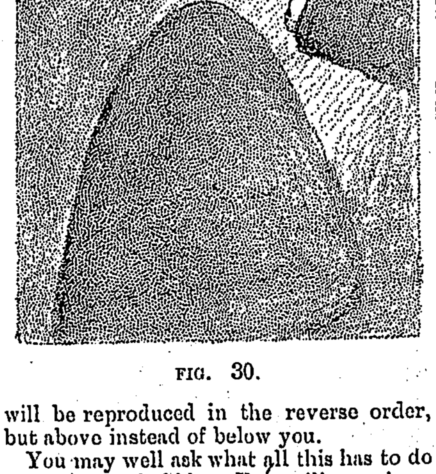

FIG. 30.

will be reproduced in the reverse order, but above instead of below you.

You may well ask what all this has to do with a soap-bubble. You will see in a moment. When you light a candle, the base of the candlestick throws the space behind it into darkness, and the form of this dark space, which is everywhere round like the base, and gets larger as you get further from the flame, is a cone, like the wooden model on the table. The shadow cast on the wall is of course the part of the wall which is within this cone. It is the same shape that you would find if you were to cut a cone through with a saw, and so these curves which I have shown you are called conic sections. You can see some of them already made in the wooden model on the table. If you look at the diagram on the wall (Fig. 31), you will see a complete cone at first upright (A), then being gradually tilted over into the positions that I have specified. The black line in the upper part of the diagram shows where the cone is cut through, and the shaded area below shows the true shape of these shadows, or pieces cut off, which are called sections. Now in each of these sections there are either one or two points, each of which is called a focus, and these are indicated by conspicuous dots. In the case of the circle (D Fig. 31), this point is also the centre. Now if this circle is made to roll like a wheel along the straight line

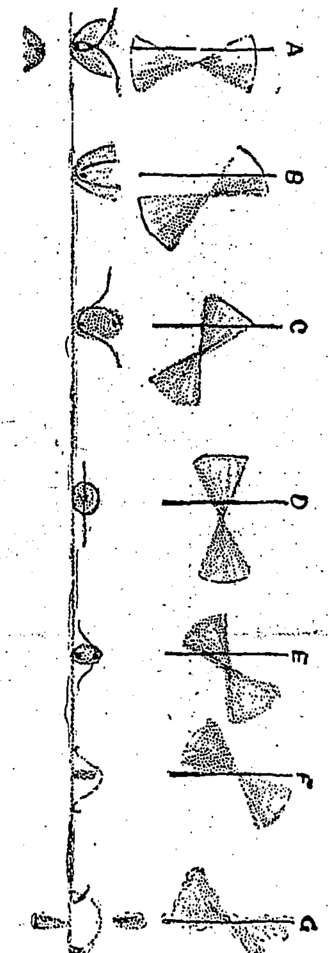

FIG. 31.

drawn just below it, a pencil at the centre will rule the straight line which is dotted in the lower part of the figure; but if we were to make wheels of the shapes of any of the other sections, a pencil at the focus would certainly not draw a straight line. What shape it would draw is not at once evident. First consider any of the elliptic sections (B, E, or F) which you see on either side of the circle. If these were wheels, and were made to roll, the pencil as it moved along would also move up and down, and the line it would draw is shown dotted as before in the lower part of the figure. In the same way the other curves, if made to roll along a straight line, would cause pencils at their focal points to draw the other dotted lines.

We are now almost able to see what the conic section has to do with a soap-bubble. When a soap-bubble was blown between two rings, and the pressure inside was varied, its outline went through a series of forms, some of which are represented by the dotted lines in the lower part of the figure, but in every case they could have been accurately drawn by a pencil at the focus of a suitable conic section made to roll on a straight line. I called one of the bubble forms, if you remember, by its name, catenoid; this is produced when there is no pressure. The dotted curve in the second figure B is this one; and to show that this catenary can be so drawn, I shall roll upon a straight edge a board made into the form of the corresponding section, which is called a parabola, and let the chalk at its focus draw its curve upon the black board. There is the curve, and it is as I said, exactly the curve that a chain makes when hung by its two ends. Now that a chain is so hung you see that it exactly lies over the chalk line.

All this is rather difficult to understand, but, as these forms which a soap-bubble takes afford a beautiful example of the most important principle of continuity, I thought it would be a pity to pass it by. It may be put in this way. A series of bubbles may be blown between a pair of rings. If the pressures are different the curves must be different. In blowing them the pressures slowly and continuously change, and so the curves cannot be altogether different in kind. Though they may be different curves, they also must pass slowly and continuously one into the other. We find the bubble curves can be drawn by rolling wheels made in the shape of the conic sections on a straight line, and so the conic sections, though distinct curves, must pass slowly and continuously one into the other. This we saw was the case, because as the candle was slowly tilted the curves did as a fact slowly and insensibly change from one to the other. There was only one parabola, and that was formed when the side of the cone was parallel to the plane of section, that is when the falling grease just touched the edge of the candlestick; there is only one bubble with no pressure, the catenoid, and this is drawn by rolling the parabola. As the cone is gradually inclined more, so the sections become at first long ellipses, which gradually become more and more round until a circle is reached, after which they become more and more narrow until a line is reached. The corresponding bubble curves are produced by a gradually increasing pressure, and, as the diagram shows, these bubble curves are first wavy (C), then they become straight when a cylinder is formed (D), then they become wavy again (E and F), and at last, when the cutting plane, *i. e.*, the black line in the upper figure, passes through the vertex of the cone the waves become a series of semicircles, indicating the ordinary spherical soap-bubble. Now if the cone is inclined ever so little more a new shape of section is seen (G), and this being rolled, draws a curious curve with a loop in it; but how this is so it would take too long to explain. It would also take too long to trace the further positions of the cone, and to trace the corresponding sections and bubble curves got by rolling them. Careful inspection of the diagram may be sufficient to enable you to work out for yourselves what will happen in all cases. I should explain that the bubble surfaces are obtained by spinning the dotted lines about the straight line in the lower part of Fig. 31 as an axis.

As you will soon find out if you try, you cannot make with a soap-bubble a great length of any of these curves at one time, but you may get pieces of any of them with no more apparatus than a few wire rings, a pipe, and a little soap and water. You can even see the whole of one of the loops of the dotted curve of the first figure (A), which is called a nodoid, not a complete ring, for that is unstable, but a part of such a ring. Take a piece of wire or a match, and fasten one end to a piece of lead, so that it will stand upright in a dish of soap water, and project half an inch or so. Hold with one hand a sheet of glass resting on the match in middle, and blow a bubble in the water against the match. As soon as it touches the glass plate, which should be wetted with the soap solution, it will become a cylinder, which will meet the glass plate in a true circle. Now very slowly incline the plate. The bubble will at once work round to the lowest side, and try to pull itself away from the match stick, and in doing so it will develop a loop of the nodoid, which would be exactly true in form if the match or wire were slightly bent, so as to meet both the glass and the surface of the soap water at a right angle. I have described this in detail, because it is not generally known that a complete loop of the nodoid can be made with a soap-bubble.

(To be Continued.)