

cement. That is to say, the water-cement ratio decreases, with a consequent increase in the strength of the concrete.

With these facts in mind, we can derive a formula for the amount of water required to give a desired consistency:—

Let W = the total quantity of water required (cu. ft.);
 C = the amount of cement required (cu. ft.);
 x = the "water factor" of the cement, or the volume

of water (cu. ft.) required to reduce 1 cu. ft. of cement to a paste;

a = the surface area of the aggregate used (sq. ft.), divided by 100;

n = the "water factor" of the aggregate, or the volume (cu. ft.) of water required to wet each 100 sq. ft. of its surface area.

Then, $W = xC + na$ (1.)

This may be simplified if na is taken to equal L , L being, therefore, the water required to wet the aggregate in any given case. The equation then becomes

$W = xC + L$ (2.)

which is the most convenient form for general use.

From Abrams' experiments we have found that the relation of compressive strength to the water and cement is given by the equation $S = A/B^R$, in which S = compressive strength in pounds per square inch; A and B are constants depending on age, materials and other conditions affecting the cement; and $R = W/C$, or "water-cement" ratio.

Substituting for W in equation 2, we have

$R = (xC + L)/C = x + L/C$ (3.)

which gives by transposition,

$L = C(R - x)$ (4.)

and

$C = L/(R - x)$ (5.)

By means of these formulas we are able to deduce several interesting conclusions which, if the premises are sound on which the formulas are based, should be capable of experimental proof. With the consistency or plasticity of a mix maintained constant, let us first determine the relationship between the compressive strength of a concrete and its cement content. In this case, since the plasticity is maintained uniform, the quantity of water required to wet the surfaces remains the same, and the only change in the quantity of water occurs in that part necessary to properly moisten the cement. Let us consider a numerical example in which the following conditions govern:—

The amount of water required to wet the cement = 22% by weight; the surface area of combined aggregate is 2,400 sq. ft.; the amount of water required to wet the aggregate is 0.75 lbs. per 100 sq. ft. of surface area of aggregate; cement proportioned on the basis of 1, 2, 3, 4 and 5 lbs. per 100 sq. ft. of surface area of aggregate; and $S = 14,000/7^R$. Our constants then become $a = 24.00$, $n = 0.75$, and therefore $L = 0.75 \times 24/62.5 = 0.288$ cu. ft.

x = weight of 1 cu. ft. of cement, multiplied by the amount of water required to wet the cement for normal consistency, divided by the weight of 1 cu. ft. of water. Therefore,

$x = 87.5 \times 0.22/62.5 = 0.308$ cu. ft.

(The standard Canadian bag of cement weighs only 87.5 lbs., but is assumed in these calculations to contain 1 cu. ft. of cement.)

C for the first case, in which the cement is proportioned 1 lb. per 100 sq. ft. of surface area, is equal to $1 \times 24/87.5 = 0.274$, and the water-cement ratio becomes

$R = 0.308 + 0.288/0.274 = 1.358$ (from Equ. 3), and

$S = 14,000/7^{1.358} = 1,074$ lbs. per sq. in.

Similarly R and S can be calculated for each of the other ratios of cement to surface area given. The result of such calculation is given in Table II.

Plotting the values of S , we obtain the curve given in Fig. 5, although this relation is not that reported by Edwards in his paper last year; but when Edwards' charts were examined after Fig. 5 was obtained, it was found that he had been misled into believing this a straight line relation by the limited range covered by his tests, and that actually four of the six sets of points shown on his charts could be better expressed by curves similar to that of Fig.

5 than by the straight lines actually shown in his paper. Figs. 2 and 3 show these.

The results of other investigators were likewise consulted and the same relation was found to hold. Among these were the results obtained by Messrs. Fuller and Thompson, some of which are shown in Fig. 4.

TABLE II.

Cement, Lbs. per 100 Sq. Ft. Surface Area.	Cement, C , in Cu. Ft.	Water-Cement Ratio, R .	Compressive Strength, S , Lbs. per Sq. In.
1	0.274	1.358	1,074
2	0.548	0.833	2,770
3	0.822	0.658	3,900
4	1.096	0.571	4,605
5	1.370	0.518	5,100

Fig. 6 shows results recently obtained in the laboratories of the Hydro-Electric Power Commission of Ontario, from a test covering the same range of proportions assumed in the calculated set of Table II.

In the case just discussed, the grading of the aggregate and the consistency of the mix were constant, and the relationship between the compressive strength of a concrete and its cement content was found. If instead, the aggregate varied but the consistency and compressive strength were to be maintained constant, how should the cement content be proportioned to bring this about?

For each strength there is a constant ratio of volume of water to volume of cement, hence the first requisite for constant strength is that this ratio be maintained constant. Secondly, with a change in grading there is a change in surface area and a corresponding change in the amount of water needed to wet the aggregate.

Let us now consider the formula $C = L/(R - x)$. For any given strength R is constant, and as long as the nature of the cement does not change, x is constant. Therefore C varies directly with L , but since $L = na$ where n is a constant for any particular aggregate, L varies directly with a , and therefore C also varies directly with a , or with the surface area of a unit volume of aggregate.

Hence to maintain a constant strength and therefore a constant R , it is necessary to proportion the cement, C , on the basis of the surface area of the aggregate.

Table III. shows in detail a number of aggregates having different mechanical analyses with their corresponding surface areas. These are shown proportioned to maintain a constant water-cement ratio, R , of 0.728, and the quantity of cement is given which is required to do this. The relation of volume or weight of cement to each 100 sq. ft. of the surface area of the aggregate used is constant, as is shown in the last column of Table III.

TABLE III.

Surface Area of Aggregate, Sq. Ft.	Cement, C , Cu. Ft.	Cement, Total Weight, Lbs.	Cement, Lbs. per 100 Sq. Ft. Surface Area.
1,600	0.457	40	2.5
2,000	0.571	50	2.5
2,400	0.686	60	2.5
2,800	0.800	70	2.5
3,200	0.914	80	2.5

Some criticism of the surface area method of proportioning has been offered because the value of the surface area obtained by Edwards' calculations was not the true surface area of the material. It was held that the aggregate particles are neither true spheres nor have they smooth surfaces, and since it is thus impossible to obtain their actual surface areas, the value for the surface area obtained is of no use.

This was given consideration, and after some experimental studies upon the uniformity of sand particles, the conclusion was reached that it is not essential that we know the exact surface area, for if we can determine a value for each case which has a constant relation to the actual