6. Show that $\frac{e}{2}=\frac{1}{\underline{|2}}+\frac{1+2}{\underline{|3}}+\frac{1+2+3}{\underline{|4}}+\ldots\ldots$ *ad inf.*

Series $e=1+1+\frac{1}{\underline{|2}}+\frac{1}{\underline{|3}}+\frac{1}{\underline{|4}}+\ldots\ldots$ *ad inf.*

$\therefore \frac{e}{2}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2\underline{|2}}+\frac{1}{2\underline{|3}}+\frac{1}{2\underline{|4}}+\ldots\ldots$ *ad inf.*......(A)

Now sum of the A. P. $1+2+3+4+\ldots\ldots n=\frac{n(n+1)}{2}$

$\therefore \frac{1+2+3+4+\ldots\ldots+n}{\underline{|n+1}}=\frac{n(n+1)}{2\underline{|n+1}}=\frac{n(n+1)}{2(n+1)n\underline{|n-1}}=\frac{1}{2\underline{|n-1}}$

Now when $n=2$, $\frac{1+2}{\underline{|3}}=\frac{1}{2}$

" $n=3$, $\frac{1+2+3}{\underline{|4}}=\frac{1}{2\underline{|2}}$

" $n=4$, $\frac{1+2+3+4}{\underline{|5}}=\frac{1}{2\underline{|3}}$, and so on.

Substituting these values in (A) we get the required expression.

7. A man has 1000 apples for sale; he sells at first so as to gain at the rate of 50% on the cost price. He sells the remainder for what he can get, losing thereby at the rate of 10%. His total gain is at the rate of 29%. How many apples did he sell at the losing rate?—*Science and Art Examination, 1882.*

Let x= number @ 50% gain, $\therefore$ $1000-x$ = number @ 10% loss.

$\therefore$ Gain on first lot $=\frac{x}{2}$ apples,

and loss on second lot $=\frac{1000-x}{10}$ apples.

$\therefore$ Total gain $=\frac{x}{2}-\frac{1000-x}{10}=29\%=290$ apples.

$\therefore$ $x=650$; remainder = 350.

8. If $z=\sqrt{(x^2+y^2)}$, show that

$x+y+z:-x+y+z::x-y+z:x+y-z.$

—*Science and Art Examination, 1881.*

$z^2=x^2+y^2$, $\therefore$ $2z^2=2x^2+2y^2$, or $z^2-x^2-y^2=x^2+y^2-z^2$

Add $2xy$ to both sides, and

$z^2-(x-y)^2=(x+y)^2-z^2$

i.e., $(z+x-y)(z-x+y)=(x+y+z)(x+a-z)$ whence, &c.

9. Solve the following sets of equations, finding all the values of x, or x and y.—*Science and Art Examination, 1881.*

(*a*) $\frac{1}{x+a}+\frac{1}{x+b}=\frac{1}{a-x}+\frac{1}{b-x}$.

Transpose, $\frac{1}{a+x}-\frac{1}{a-x}=\frac{1}{b-x}-\frac{1}{b+x}$

i.e., $\frac{-2x}{a^2-x^2}=\frac{2x}{b^2-x^2}$ $\therefore$ $x=0$

$\therefore$ $-b^2+x^2=a^2-x^2$ $\therefore$ $x=\pm\sqrt{\left(\frac{a^2+b^2}{2}\right)}$.

(*b*) $x^2+y=51$ and $2x^2+y^2=102$.

Multiply (1) by 2 and subtract $\therefore$ $y^2-2y=0$ $\therefore$ $y=0$ or 2.

Substitute in (1) and $x=\pm 7$ or $\pm\sqrt{51}$.

(*c*) $\sqrt{(x+y)}+\sqrt{(x-y)}=5$, and $\sqrt{(x^2-y^2)}=4{\cdot}5$

Square 1st and subtract twice 2nd and $2x=16$, $x=8$

Square 2nd and substitute, and $y=\pm 6{\cdot}6143\ldots\ldots$

10. In a field which grows uniformly 31 oxen can consume $8\frac{3}{4}$ acres in $\frac{2}{3}$ of the time in which 15 oxen would consume $5\frac{1}{4}$ acres, and 22 oxen would require 3 days longer to consume $7\frac{1}{2}$ acres than 20 oxen would require for $6\frac{1}{4}$ acres: in what time would 31 oxen eat up the $8\frac{3}{4}$ acres?—*Colenso's Arithmetic.*

acres.		acres.		oxen.		oxen.	
$5\frac{1}{4}$	:	$8\frac{3}{4}$	=	15	:	25	(1)
$7\frac{1}{2}$	:	$8\frac{3}{4}$	=	22	:	$25\frac{2}{3}$	(2)
$6\frac{1}{4}$	:	$8\frac{3}{4}$	=	20	:	28	(3)

Let u= a certain unit of time: put $4u$= time required by 25 oxen to eat the grass on $8\frac{3}{4}$ acres, $\therefore$ $3u$= time required by 31 oxen.

Then $25:31=3u:3\frac{18}{25}u$;

And $4u-3\frac{18}{25}u=\frac{7}{25}u$ = growth of grass eaten by 25 oxen in one unit of time.

$\frac{7}{25}u:4u=u:14\frac{2}{7}u$ growth.

$\therefore$ $14\frac{2}{7}u-4u=10\frac{2}{7}u$= original quantity of grass.

Now 25 oxen in $\frac{7}{25}u$ eat 1 u growth,

$\therefore$ 25 " 1 u " $\frac{25}{7}u$ "

$\therefore$ 1 " 1 u " $\frac{1}{7}u$ "

And $25\frac{2}{3}$ " 1 u " $3\frac{2}{3}u$ " from (2)

And 28 " 1 u " 4 u " " (3)

That is 28 oxen eat $(4u-u)=3u$ of the original growth,

and $25\frac{2}{3}$ " $(3\frac{2}{3}u-u)=2\frac{2}{3}u$ " " "

$\therefore$ $25\frac{2}{3}$ oxen would eat the whole original grass in $10\frac{2}{7}\div 2\frac{2}{3}=3\frac{6}{7}u$,

and 28 " " " " " $10\frac{2}{7}\div 3=3\frac{3}{7}u$.

$\therefore$ The given difference of 3 days $=\frac{3}{7}u$.

But 31 oxen require $3u$ to eat the grass,

$\therefore$ $\frac{3}{7}u:3u=3$ days : 21 days, the time required.

11. If the series of natural numbers 1, 2, 3 ... 10, 11, 12...... were written down in a row without separating the figures, what would be the 750th figure of the row?—*Bursary Competition, Aberdeen University.*

Up to 99 there are $9\times 10\times 2+9=189$ figures, and the three figure numbers commence. We require $750-189=561$ figures more.

$561\div 3$ gives 187 numbers of 3 figures each. Hence the last number will be $99+187=286$, and the last figure is 6.

12. Given $(x+y)^{\frac{1}{3}}+(x-y)^{\frac{1}{3}}=a^{\frac{1}{3}}$ (1)
and $(x^2+y^2)^{\frac{1}{3}}+(x^2-y^2)^{\frac{1}{3}}=a^{\frac{1}{3}}$ (2), find x and y.

—*St. John's College, Cambridge.*

Cube (1) by formula $(a+b)^3=a^3+b^3+3ab(a+b)$ and substitute for $a+b$, and we have

$$2x+3a^{\frac{1}{3}}(x^2-y^2)^{\frac{1}{3}}=a,\quad \therefore\ x^2-y^2=\frac{(a-2x)^3}{27a}\qquad (A).$$

Then from (2) $(x^2+y^2)^{\frac{1}{3}}+\frac{a-2x}{3a^{\frac{1}{3}}}=a^{\frac{1}{3}}$.

Cubing as above $x^2+y^2=\frac{(2a+2x)^3}{27x}$ (B).

(A) + (B) gives $54ax^2=(2a+2x)^3+(a-2x)^3$
$=9a(a^2+2ax+4x^2)$

$\therefore$ $x^2-ax=\frac{1}{2}a^2$, whence $x=\frac{1}{2}a^2(2\pm\sqrt{3}$.

Substitute for x in (B) and

$$\tfrac{1}{2}a^2(2\pm\sqrt{3})+y^2=\frac{a^2(18\pm 10\sqrt{3})}{9}$$

$$\therefore\ y=\tfrac{1}{3}a\sqrt{(36\pm 22\sqrt{3})}$$

$$=\frac{a}{2}\left(1-\frac{1}{\sqrt{3}}\right)\sqrt{\left(1-\frac{4}{\sqrt{3}}\right)}$$

ONTARIO EDUCATION DEPARTMENT.

SECOND CLASS TEACHERS, JULY, 1883.

1. Prove that $\frac{1}{4}$ of $\frac{3}{7}=\frac{3}{28}$.

Let $\frac{1}{4}$ of $\frac{3}{7}$ = value.

$\therefore$ the whole of $\frac{3}{7}$ = value $\times 4=\frac{1}{7}+\frac{1}{7}+\frac{1}{7}$

$\therefore$ value $\times 28=1+1+1=3$

$\therefore$ value $=3\div 28=\frac{3}{28}$.

N.B.—In this proof we have assumed multiplication and division of equals by the same, $\frac{4}{4}$ or $\frac{7}{7}=1$, and that a fraction expresses the division of numerator by denominator.

Simplify $(2\frac{2}{7}$ of $3\frac{1}{16})+\frac{4}{5}-(1\frac{1}{3}$ of $1\frac{5}{16})-(1\frac{3}{4}$ of $4\frac{4}{5}$ of $\frac{5}{12})$

$=7+\frac{4}{5}-7-\frac{12}{7}=-1\frac{11}{35}$.

2. The pendulum of one clock makes 24 beats in 26 seconds; that of another, 36 beats in 40 seconds. If they start at the same time, when first will the beats occur together?

1st will make its 120th and 2nd its 117th beat at the end of the 130th second. Answer, 2′ 10″.

3. *A* can do as much work in 4 hours as *B* can in 6; *B* in $3\frac{1}{2}$ as

6)130 13/12 136 9/11 10/9