2. Express $m^4-4m^3n+5m^2n^2-2mn^3+n^4$ as a rational integral function of p and n, where $p=m-n$.

3. If y is a rational integral function of x, and y becomes zero when a is substituted for x, prove that $x-a$ is a factor of y.

Resolve into factors—

$$x^3-\{a(a-b)+b(b-c)+c(c-a)\}x+\{ab(a-b)+bc(b-c)+ca(c-a)\}$$

4. If $\frac{a}{b}=\frac{c}{d}$ prove that $\frac{ma+nb}{ma-nb}=\frac{mc+nd}{mc-nd}$.

If $$\frac{x^2+\left\{\frac{m+n}{m-n}+\frac{m-n}{m+n}\right\}xy+y^2}{x^2+\left\{\frac{m+n}{m-n}-\frac{m-n}{m+n}\right\}xy-y^2}=\frac{v+1}{v-1}$$

find the simplest expression for the value of v.

5. What is a ratio? Does the ratio of two quantities depend upon their magnitude?

Given $y^4+x^2y-a^2x^2=0$, to find the ratio of x to y when x becomes indefinitely great.

6. What is meant by a *maximum* or *minimum* solution?

It is required to divide a number a into two parts such that the quotient arising from dividing their product by the sum of their squares may be a minimum. Determine the quotient, and the division of the number required to produce it.

7. In an arithmetic series, find an expression giving the last term in terms of the first term the common difference and the sum of the series.

The nth terms of two *A.P.'s* are respectively $\frac{1}{2}(n+2)$ and $\frac{1}{2}(3n-1)$. The same number of terms being taken in each series, what is the number when the sum of the second series is four times that of the first?

What is the *greatest* ratio of the sum of any number of terms of the second series to the sum of the same number of terms of the first?

8. The attraction of a planet upon a body at its surface varies directly as the planet's mass and inversely as the square of its radius. The length of a pendulum varies directly as the attraction and inversely as the square of the number of beats which it makes in a given time. The mass of the earth being 75 and of the moon 1, the radius of the earth 4,000 miles and of the moon 1,100, and the length of a pendulum which beats 5 times in 2 seconds at the earth's surface being 6.26 in., find the length of a second's pendulum at the moon's surface.

9. From a company of 15 men 6 are selected each night as a guard. How often, respectively, will A and B be together (1) with C? (2) without C? (3) with C or D? (4) with C and D?

10. Given $x^2+\frac{a-b}{ab}x+\frac{ab}{a-b}=0$.

(1) Express b in terms of a when the two values of x are (α) equal in magnitude and opposite in signs; (β) equal in magnitude and of like signs.

(2) If x_1 x_2 be the roots, express the value of

$$\frac{1}{x_1}+\frac{1}{x_2} \text{ in terms of } a \text{ and } b.$$

SOLUTIONS—BY OUR CORRESPONDENT D.

1. A rational binomial factor is of the form $x+a$, and if several such be multiplied together, as $(x+a)(x+b)(x+c)\ldots$, the independent term will consist of the constant product $abc\ldots$.

$\therefore$ In our search for factors, we employ only factors of the independent term.

Again, if the whole expression becomes zero, one of the factors at least, $x+a$ say, must be zero, and therefore $x=-a$.

Hence a quantity which when substituted for x in the given expression renders its value zero, is itself the second term, with sign changed, of a binomial factor.

Example—The factors of 24 are ±1, ±2, ±3, ±4, ±6, ±12, ±24; and upon trial the expression vanishes when -1, -2, 3 or 4 is substituted for x.

$\therefore$ $x+1$, $x+2$, $x-3$, $x-4$ are four binomial factors.

2. Since the required expression is to be a function of p and n only, m must not appear. Now $m=p+n$, and writing this value for m, we have

$$(p+n)^4-4(p+n)^3n+5(p+n)^2n^2-2(p+n)n^3+n^4;$$

which being expanded and reduced, give, $p^4-p^2n^2+n^4$, as the required expression.

3. Let $y=px^n+qx^{n-1}+\ldots ux+v$.

Then by the question, $o=pa^n+qa^{n-1}+\ldots ua+v$.

Subtracting the second of these expressions from the first,

$$y=p(x^n-a^n)+q(x^{n-1}-a^{n-1})+\ldots u(x-a)$$

an expression which is divisible by $x-a$, since every term is divisible by that quantity,

$\therefore$ y is divisible by $x-a$.

Example—This expression is of the form x^3+ox^2+qa+x,

Where $q=-a(a-b)-b(b-c)-c(c-a)$,

$=ab-a^2+bc-b^2+ca-c^2$,

and $v=ab-(a-b)+bc(b-c)+ca(c-a)$.

Substituting b for a in the expression for v it becomes zero;

$\therefore$ $a-b$, and from symmetry $b-c$, $c-a$, are factors of v, and we readily find, $v=-(a-b)(b-c)(c-a)$.

If the expression can be factored, then, the factors are of the form $x-(a-b)$, $x-(b-c)$, and $x-(c-a)$; and since

$(a-b)+(b-c)+(c-a)=0$,

and $(a-b)(b-c)+(b-c)(c-a)+(c-a)(a-b)=9$,

$\therefore$ $\{x-(a-b)\}\{x-(b-c)\}\{x-(c-a)\}$ are the factors req'd.

4. Since $\frac{a}{b}=\frac{c}{d}$ $\therefore$ $\frac{a}{c}=\frac{b}{d}$.

Let $\frac{a}{c}=\frac{b}{d}=z$.

Then $a=cz$ and $ma=mcz$,

$b=dz$ and $nb=ndz$.

Adding and subtracting,

$ma+nb=(mc+nd)z$,

$ma-nb=(mc-nd)z$;

$\therefore$ by division,

$$\frac{ma+nb}{ma-nb}=\frac{mc+nd}{mc-nd}$$

(2) The first fraction is evidently,

$$\frac{\left(x+\frac{m+n}{m-n}\cdot y\right)\left(x+\frac{m-n}{m+n}\cdot y\right)}{\left(x+\frac{m+n}{m-n}\cdot y\right)\left(x-\frac{m-n}{m+n}\cdot y\right)}$$

Since the product of the coefficients of y is unity, and their sum or difference is the coefficient of xy,

$$\therefore \quad \frac{x+\frac{m-n}{m+n}\cdot y}{x-\frac{m-n}{m+n}\cdot y}=\frac{v+1}{v-1}$$

Adding and subtracting numerators and denominators,

$$\frac{x}{\frac{m-n}{m+n}\cdot y}=v=\frac{x}{y}\cdot\frac{m+n}{m-n}.$$

5. The ratio of one quantity to another is the quotient arising from dividing the former quantity by the latter.

(This being an Algebra paper the definition of a geometrical ratio is not required.)

Ratio depends upon the *relative* but not upon the *absolute* magnitudes of the quantities concerned.

Example—Divide through by x^4,

Then $\frac{y^4}{x^4}+\frac{y}{x}-\frac{a^2}{x^2}=0$.

Now a being finite while x becomes indefinitely great, $\frac{a^2}{x^2}$ becomes indefinitely small and may be rejected.

$$\therefore \quad \frac{y^4}{x^4}=-\frac{y}{x} \text{ when } x=\infty.$$

Divide throughout by $\frac{y}{x}$,

Then $\frac{y^3}{x^3}=-1$, and $\frac{y}{x}=0$, (since we div by it) when $x=\infty$;

$\therefore$ $\frac{y}{x}=\sqrt[3]{-1}$, one root of which is -1, and also $\frac{y}{x}=0$.