Trisection of a Rectilineal Angle.

Let B O C be the angle to be trisected.

At the point O, make (23.1) the angles B O D and C O E, each equal to the angle B O C. Take O as centre with any radius O B, and describe the arc D B C E; join D B, B C, C E, and D E. Produce D B, and E C to meet in T; join OT. By 4.1., the angle ODB is equal to the angle O E C, and (5.1) the angle O D E is equal to the angle O E D; take away the two latter angles and the remaining angles E D T and D E T are equal: therefore (6.1) D T is equal to E T. In the triangles O D T and O E T, the three sides of the one are respectively equal to the three sides of the other, by (8.1) they are equal in every respect; therefore the angle D T O is equal to the angle E T O, then the line O T bisecting the vertical angle of an isosceles triangle, bisects D E and B C perpendicularly. It also bisects the arc BC; it is evident by 4.1 that DB, BC, and C E are equal to one another.

Draw a d parallel to B C. so that a d shall be equal aB and d C. This is done by bisecting the angles T B C and T C B by lines meeting the sides B T and C T in a and d, when the triangle is isosceles, or equilateral and proved by (29.1, 6.1, and 4.1). Find the diamater of a circle circumscribing the trapizium B a d C, and make t L equal to it; join a L and d L, cutting the arc B C in the points r and l: these are the trisecting points of the arc B C which is t measure of the given angle B O C. Draw the lines O r b ard O l c, and the three angles b O c, b O B, and C O c are equal and each equal to one third of the given angle B O C.