The economic drop of voltage per mile of single conductor is given by the formula:[1]

$$e_r = 8.1\sqrt{\frac{a \times p}{p_1}}$$

Where p is the price in dollars of 100 lb. weight of conductor (in this example $p = 15$), a is the percentage to cover annual depreciation and interest on cost of conductors, and p_1 is the cost per horse-power-year of the wasted power. The proper value for a may be arrived at by estimating the term of years corresponding to the life of the conductors, at the end of which they are supposed to be of no value. Taking 20 years as the life of the conductors the depreciation to be allowed according to the table herewith is 3.03, which makes $a = 6 + 3.03$, or, say, 9 per cent.

With regard to p_1, if the demand for power were equal to the available supply from the time of the power-plant being put into operation, the works cost of waste power would be the same as the selling price; but, on the assumption that the supply exceeds the demand during the first five years of operation, and that the cost of waste power during this period is only $7 per horse-power-year.[2]

The average cost of wasted power during the 20 years life of the conductors is:

$$p_1 = \frac{(5 \times 7) + (15 \times 21)}{20} = \$17.50$$

The economic voltage drop is therefore:

$$e_r = 8.1 \times \sqrt{\frac{9 \times 15}{17.5}} = 22.5 \text{ volts per mile.}$$

A first approximation to the required line voltage may be obtained by the formula

$$V_k = 5.5\sqrt{\text{distance} + \frac{\text{horsepower}}{200}}$$

$$= 5.5\sqrt{50 + \frac{15.000}{200}}$$

$$= 61.5, \text{ or say } 60{,}000 \text{ volts.}$$

In order to calculate the cost of the line losses, it will be necessary to adopt a figure for the horse-power transmitted, which, when squared, will give the average square of the power during the estimated life of the conductors.

Assuming an average figure for the output during 12 months, a table showing probable demand for power can be constructed as follows:

Period.	Hp. Demanded.	Hp. Squared by Years.
First year of working	5 by 1000	25 by 10^6 by 1 = 25 by 10^6
Second year of working	6 by 1000	36 by 10^6 by 1 = 36 by 10^6
Third year of working	7 by 1000	49 by 10^6 by 1 = 49 by 10^6
Fourth year of working	9 by 1000	81 by 10^6 by 1 = 81 by 10^6
Fifth year of working	12 by 1000	144 by 10^6 by 1 = 144 by 10^6
Remaining 15 years of estimated life of conductors	15 by 1000	225 by 10^6 by 15 = 3375 by 10^6

Total of last column, 3710 by 10^6.

Average, 185.5 by 10^6.

Average hp. for purpose of calculating cost of waste power = $\sqrt{185.5 \text{ by } 10^6}$ = 13,600 approximately.

When the section of the conductors is such as to satisfy Kelvin's law of economy, the yearly cost of the I^2R losses is equal to the amount representing annual depreciation and interest on first cost of conductors; and the total annual charges on active line material will therefore be:

$$2 \times \frac{I^2R \times p_1}{746} \times 3 \times 1$$

where R is the resistance per mile of conductor. But

$$I = \frac{P \times 746}{\sqrt{3} \times E \times \cos\theta}$$

where P stands for the horse-power transmitted.

Also: IR = voltage loss per mile = e_r. So that the formula for the total yearly charges on conductors can be written

$$\frac{2 \times \sqrt{3} \times e_r \times P \times p_1 \times 1}{E \times \cos\theta}$$

which in this example becomes:

$$\frac{2 \times \sqrt{3} \times 22.5 \times 13{,}600 \times 17.5 \times 50}{60{,}000 \times 0.8} = \$19{,}300$$

Closer Estimate of Economic Voltage.—In order to take into account first cost, life, annual maintenance, and operating charges of every portion of the complete undertaking which may be affected by a change in the transmission voltage, the costs, worked out on an annual basis, may be arranged in tabular form as here shown, where the total charges for the 60,000-volt scheme are compared with the estimated charges for an 80,000-volt transmission. In this particular example, the figures are favorable to the higher voltage; but the difference is small. It would be useless to repeat the process for a voltage lower than 60,000, because the cost would certainly be higher.

Comparison of Costs at Different Voltages.

Portion of Complete Undertaking Affected by Change of Voltage:	Estimated life (yr.)	Depreciation (from tables)	Depreciation plus 6 per cent. int.	Total Cost. Voltage 60,000	Total Cost. Voltage 80,000	Annual Charges. Voltage 60,000	Annual Charges. Voltage 80,000
Line conductors (copper) of most economic section (annual cost varies as $\frac{1}{\text{voltage}}$)	20	...		......	......	$19,300	$14,475
Steel tower transmission-line, without conductors, but otherwise complete (from curves, Fig. 2)	18	3.55	9.55	$108,000	$130,000	10,310	12,410
Generating-station buildings	40	0.828	6.828	56,600	57,000	3,870	3,895
Sub-station buildings	30	1.505	7.505	11,500	11,900	863	893
Transformers	18	3.55	9.55	32,500	35,200	3,100	3,360
Switch-gear, including lightning arresters, cables in buildings, and entering bushings	14	5.10	11.10	27,000	34,000	3,000	3,775
Assume unaltered: Yearly cost of power lost in generators and transformers. Yearly cost of operation and maintenance. Right-of-way and clearing.						$40,443	$38,808

Difference in favor 80,000 volts = $1635.

It will be understood that the accompanying estimate of total annual charges of the two selected voltages does not include any items other than those that are liable to vary with changes in the line voltage. An estimate covering the complete undertaking would, in addition to the items named, have to take account of riparian rights for dam, reservoir, etc., preliminary legal and other expenses; cost of providing proper access for materials to site of works; dam and hydraulic works outside station building; turbines; electric generators and exciters; auxiliary plant; sundries and contingencies.

In the case of a short distance transmission with a line pressure not exceeding 11,000 volts, and the possibility of winding the generators for the full pressure, the relative costs and efficiencies of generators wound for different voltages should be taken into account.

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[1] This and the one or two subsequent formulas are either taken from, or suggested by, an article by me which appeared in the Electrical World of September 23, 1911, and in which their derivation is explained.

[2] The actual works costs of the wasted power is always difficult to determine exactly. It must, however, be remembered that even with unlimited power, and no appreciable increase in maintenance and operating charges with increase of losses, the greater capital cost of the plant installed to provide this waste power has to be taken into account and expressed in the form of an annual charge per horsepower wasted, whether this waste occurs in the generating and transforming plant or the line itself.