MISCELLANEOUS ARTICLES AND EXERCISES.

1. Prove the following relations:

(a) $\cos^4 A-\sin^4 A=2\cos^2 A-1.$

(b) $\sqrt{1-\sin\theta}=(\sec\theta-\tan\theta)\sqrt{1+\sin\theta}.$

(c) $2\sec^2\theta=\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta-1}+\dfrac{\operatorname{cosec}\theta}{\operatorname{cosec}\theta+1}.$

(d) $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A}=\sin A+\cos A.$

(e) $\sec^4 A-\sec^2 A=\tan^4 A+\tan^2 A.$

(f) $\sin A+\sin B-\sin C=4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$ in any triangle.

2. Find any function of θ from the following:

(a) $2\sin\theta=2-\cos\theta$

(b) $8\sin\theta=4+\cos$

(c) $\tan\theta+\sec\theta=3/$[illegible].

(d) $\sin\theta+2\cos\theta=1.$

(e) $\tan 2\theta+\cot\theta=8\cos^2\theta.$

3. In any circle prove that the chord of 108° is equal to the sum of the chords of 36° and 60°.

4. A person on a light-house notices that the angle of depression of a boat coming towards him is α, and that after m minutes it is β. How long after the first observation will the boat reach the light-house?

5. (a) From the cosine formula show that

$$c=(a+b)\sin\frac{C}{2}\sec\varphi,\text{ if }\tan\varphi=\frac{a-b}{a+b}\cot\frac{C}{2}.$$

(b) Express the results of (a) in logarithmic form, and apply it to the case where $a=25.33$, $b=18.46$, $C=78°\,44'$.

6. Prove that $a\cos\theta+b\sin\theta$

$$=\sqrt{a^2+b^2}\cos\left[\theta-\tan^{-1}\frac{b}{a}\right].$$

7. Show from Ex. 6, that $a\cos\theta+b\sin\theta$ is a maximum when $\theta=\tan^{-1}b/a$.

8. (a) Divide analytically the angle A into two parts such that the sum of the cosines of the parts is a given quantity m.