V. If $ax^4+bx^3+cx^2+dx^2+8$ be a perfect square, it must be of the form $(mx^2+nx+p)^2$
$= m^2x^4+2mnx^3+\overline{n^2+2mp}\,x^2+2npx+p^2.$

Equating coefficients of like powers of x:

$$a=m^2, \qquad d=2np,$$
$$b=2mn, \qquad e=p^2.$$
$$c=m^2+2mp,$$

Then $\dfrac{a}{8}=\dfrac{m^2}{p^2}$ and $\dfrac{b^2}{a^2}=\dfrac{4m^2n^2}{4n^2p^2}=\dfrac{m^2}{p^2}$,

and $\dfrac{b^2}{4a}+\dfrac{2ad}{b}=\dfrac{4m^2n^2}{4m^2}+\dfrac{4m^2np}{2mn}$,
$$=n^2+2mp$$
$$=c,$$

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IV. (a.) If
$$x+y+z=1+\sqrt{2(1-x)(1-y)(1-z)},$$
shew that $x^2+y^2+z^2+2xyz=1$.

By squaring, the expression becomes
$$x^2+y^2+z^2=$$
$$1+2\left\{1-(x+y+z)+\sqrt{2(1-x)(1-y)(1-z)}\right\}$$
$-2xyz$; that is, $x^2+y^2+z^2+2xyz=1$, since
$$2\left\{1-(x+y+z)+\sqrt{2(1-x)(1-y)(1-z)}\right\}=0.$$

(c.) (1) $x=y+\dfrac{1}{z}$, (2) $y=z+\dfrac{1}{x}$, shew that $z=x-\dfrac{2}{y}$.

By (1) $z=\dfrac{1}{y-z}$; by (2) $z=\dfrac{xy-1}{x}$.

$$\therefore \frac{1}{x-y}=\frac{xy-1}{x}=z;$$

$$\therefore z=\frac{xy-1-1}{x-x+y}=\frac{xy-2}{y}=x-\frac{2}{y}.$$

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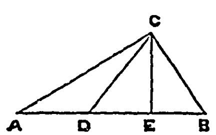
ANOTHER SOLUTION OF II.



II. Let AB be trisected at D and E. Join CD and CE. Then

$$AC^2+CE^2=2AD^2+2CD^2,$$
$$\therefore CD^2=\frac{AC^2}{2}+\frac{CE^2}{2}-AD^2. \qquad (1)$$
$$CD^2+BC^2=2AD^2+2CE^2,$$
$$\therefore CE^2=\frac{CD^2}{2}+\frac{BC^2}{2}-AD^2 \qquad (2)$$

Substituting from (2) in (1),
$$CD^2=\frac{AC^2}{2}-AD^2+\frac{CD^2}{4}+\frac{BC^2}{4}-\frac{AD^2}{2},$$

$$\therefore \frac{3CD^2}{4}=\frac{AC^2}{2}-\frac{3AD^2}{2}+\frac{BC^2}{4},$$
$$\therefore \frac{3CD^2}{4}=\frac{AC^2}{2}-\frac{3AD^2}{2}+\frac{9AD^2}{4}-\frac{AC^2}{4},$$
$$\therefore CD^2=\tfrac{1}{3}AC^2+AD^2.$$

VII. $\sin 3A=\cos 2A$, $\therefore 90-2A=3A$,
$$\therefore \cos(90-2A)=\cos 3A,$$
$$\therefore 90-2A=n.360^\circ\pm 3A.$$
$$\therefore A=\frac{90-n.360^\circ}{2\pm 3}.$$

Where n is an integer, negative or positive,

Put $n=\ \ 0$ $\therefore A=18^\circ$,
" $n=+1$ $\therefore A=270^\circ$,
" $n=-1$ $\therefore A=90^\circ$,
" $n=-2$ $\therefore A=162^\circ$,
" $n=-3$ $\therefore A=234^\circ$,
" $n=-4$ $\therefore A=306^\circ$,

Since we are to take only those values of A which are positive, and less than 360°.

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IV. (b.) If $a^2+b^2+c^2=2S^2$, then
$$(S^2-a^2)(S^2-b^2)+(S^2-c^2)(S^2-a^2)+(S^2-b^2)(S^2-c^2)$$
$$=\frac{(b^2+c^2-a^2)(a^2+c^2-b^2)}{4}+\frac{(a^2+b^2-c^2)(b^2+c^2-a^2)}{4}+\frac{(a^2+c^2-b^2)(a^2+b^2-c^2)}{4}. \qquad (1)$$

Again, if $a+b+c=2s$,
$$4s(s-a)(s-b)(s-c)=$$
$$\frac{(a+c+b)(c+b-a)(a+c-b)(a+b-c)}{4}. \qquad (2)$$

First, to prove that (2) is a factor of (1).

If $(a+b+c)$ is a factor of (1), then by putting $a+b+c=0$ in (1), that expression ought to vanish. Upon doing this the expression becomes
$$\frac{4abc^2+4acb^2+4bca^2}{4}=abc(a+b+c)=0.$$

$\therefore a+b+c$ is a factor of (1); similarly the other three factors of (2) may be shewn to be factors of (1). Hence (2) is a factor of (1).

Next, to prove that (1) = (2).

Since (2) is a factor of (1), we may put (1) = $N\times$ (2) where N is to be found and does not depend upon the factors in (2); thus we can give any values we please to the letters involved.