UNIVERSITY OF TORONTO.

ANNUAL EXAMINATIONS—1880.

PROBLEMS.

Honours.

1. Find a point within an isosceles triangle such that its distance from each of the base angles is half its distance from the vertical angle.

2. If an exterior angle of a triangle be bisected by a straight line which likewise cuts the base; the rectangle contained by the sides of the triangle, together with the square on the line bisecting the angle is equal to the rectangle contained by the segments of the base.

3. If x, y, z, be the perpendiculars from the angles of a triangle on the opposite sides, and if

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{2}{\sigma},$$

prove that

$$4\sqrt{\frac{1}{\sigma}\left(\frac{1}{\sigma}-\frac{1}{x}\right)\left(\frac{1}{\sigma}-\frac{1}{y}\right)\left(\frac{1}{\sigma}-\frac{1}{z}\right)} = \frac{1}{\text{area of triangle}}.$$

IV. Prove that every power of the sum of two squares may be divided into two parts, each of which is the square of an integer.

V. Find the sum of the series

$$\frac{4}{1\cdot 5}+\frac{9}{5\cdot 14}+\frac{16}{14\cdot 30}+\frac{25}{30\cdot 55}+\ldots\ldots \text{ to } n \text{ terms},$$

the last factor in the denominator being the sum of the other factor and the numerator.

VI. If n be prime, prove that any number in the scale whose radix is $2n$ ends in the same digit as its n^{th} power.

VII. If

$\frac{p_r}{q_r}$ be the r^{th} convergent to $\frac{\sqrt{5}+1}{2}$,

prove that

$$p_3 + p_5 + \ldots + p_{2n-1} = p_{2n} - p_2,$$
$$q_3 + q_5 + \ldots\ q_{2n-1} = q_{2n} - q_2,$$

VIII. Find the number of combinations that can be made out of the letters in the following line:

απαππαπαῖ, παπαππαπαππαπαππαπαῖ.

taking them (1) 5 together, (2) 25 together.

IX. If $\phi(r) = \lfloor n$

$$\left\{\frac{1}{\lfloor r\ \lfloor n-r} + \frac{1}{\lfloor r-1\ \lfloor n-r+1}\cdot r + \frac{1}{\lfloor r-2\ \lfloor n-r+2}\ \frac{(r-1)(r-2)}{1\cdot 2} + \ldots\right\}$$

prove that

$$2\left[\phi(0)+\phi(1)+\ldots+\phi(n-1)\right]+\phi(n)=3^n.$$

X. Eliminate x, y, z from the simultaneous equations.

$$\left\{\begin{aligned} \frac{\alpha}{x} &= \frac{1}{y} + \frac{1}{z} \\ \frac{\beta}{y} &= \frac{1}{z} + \frac{1}{x} \\ \frac{\gamma}{z} &= \frac{1}{x} + \frac{1}{v} \end{aligned}\right.$$

Why are these three equations sufficient for the elimination of the three unknowns?

11. If $A + B + C = \frac{\pi}{2}$, shew that

(1) $\cot A + \cot B + \cot C = \cot A \cot B \cot C$.

(2) $\tan A + \tan B + \tan C = \tan A \tan B \tan C + \sec A \sec B \sec C$.

12. ABC is an equilateral triangle; circles are described on AB and AC as diameters; tangents are drawn through the points B and C. Prove that the radius of the circle touching these tangents and the two circles is very nearly one-eighth of the side of the triangle.

13. On the side BC of the triangle ABC are drawn two equilateral triangles, $A'BC$ and $A''BC$; likewise, the equilateral triangle $B'CA$, $B''CA$ and $C'AB$, $C''AB$ are drawn on the sides CA and AB respectively. Prove that

$$A'A\cdot AA'' = B'B\cdot BB'' = C'C\cdot CC''.$$

XIV. If (p, q, r) be the perpendiculars on