where k_1 is a rational function of w, and k_c is what k_1 becomes by changing w into w^c. By putting $c=1$ in (85),

$$R_1^{\frac{1}{n}} = A_1(\phi_\sigma^\sigma \psi_\tau^\tau \ldots . X_\delta^\delta F_\beta^\beta)^{\frac{1}{n}}.$$

Taking this in connection with the second of equations (87),

$$(R_v R_1^{-v})^{\frac{1}{n}} = w^a (A_v A_1^{-v}) Q (F_\beta^{-v\beta} X_\delta^{-v\delta} \ldots .)^{\frac{1}{n}} \{(\phi_{v\sigma}^\sigma \phi_\sigma^{-v\sigma})(\psi_{v\tau}^\tau \psi_\tau^{-v\tau}) \ldots .\}^{\frac{1}{n}}. \quad (89)$$

In like manner, by putting c for e in (85), and taking the result in connection with the first of equations (87),

$$(R_{cv} R_c^{-v})^{\frac{1}{n}} = w^r (A_{cv} A_c^{-v}) Q (F_{c\beta}^{-v\beta} \ldots .)^{\frac{1}{n}} \{(\phi_{cv\sigma}^\sigma \phi_{c\sigma}^{-v\sigma})(\psi_{cv\tau}^\tau \psi_{c\tau}^{-v\tau}) \ldots .\}^{\frac{1}{n}}. \quad (90)$$

From (89) compared with the first of equations (88), and from (90) compared with the second of equations (88),

$$\left.\begin{aligned} k_1 &= w^a (A_v A_1^{-v}) Q (F_\beta^{-v\beta} \ldots .)^{\frac{1}{n}} \{(\phi_{v\sigma}^\sigma \phi_\sigma^{-v\sigma})(\psi_{v\tau}^\tau \psi_\tau^{-v\tau}) \ldots .\}^{\frac{1}{n}} \\ \text{and} \qquad k_c &= w^r (A_{cv} A_c^{-v}) Q (F_{c\beta}^{-v\beta} \ldots .)^{\frac{1}{n}} \{(\phi_{cv\sigma}^\sigma \phi_{c\sigma}^{-v\sigma})(\psi_{cv\tau}^\tau \psi_{c\tau}^{-v\tau}) \ldots .\}^{\frac{1}{n}} \end{aligned}\right\} \quad (91)$$

By § 9, because ϕ_σ is of the same structure as the expression (8),

$$(\phi_{v\sigma} \phi_\sigma^{-v})^{\frac{1}{s}} = q_\sigma,$$

q_σ being a rational function of the primitive s^{th} root of unity w^σ. And, since it appeared from the reasoning in § 9 that the nature of the function does not depend on the particular primitive s^{th} root of unity denoted by w^σ, we have at the same time

$$(\phi_{cv\sigma} \phi_{c\sigma}^{-v})^{\frac{1}{s}} = q_{c\sigma},$$

$q_{c\sigma}$ being what q_σ becomes when w is changed into w^c. Therefore, because $s\sigma = n$,

$$(\phi_{v\sigma}^\sigma \phi_\sigma^{-v\sigma})^{\frac{1}{n}} = q_\sigma$$

and

$$(\phi_{cv\sigma}^\sigma \phi_{c\sigma}^{-v\sigma})^{\frac{1}{n}} = q_{c\sigma}.$$

Similarly,

$$(\psi_{v\tau}^\tau \psi_\tau^{-v\tau})^{\frac{1}{n}} = q'_\tau$$

and

$$(\psi_{cv\tau}^\tau \psi_{c\tau}^{-v\tau})^{\frac{1}{n}} = q'_{c\tau},$$

where q'_τ is a rational function of w^τ, and $q'_{c\tau}$ is what q'_τ becomes when w is changed into w^c. Therefore, from (91),

$$\left.\begin{aligned} k_1 &= w^a (A_v A_1^{-v}) Q (F_\beta^{-v\beta} \ldots .)^{\frac{1}{n}} (q_\sigma q'_\tau \ \ldots .) \\ \text{and} \qquad k_c &= w^r (A_{cv} A_c^{-v}) Q (F_{c\beta}^{-v\beta} \ldots .)^{\frac{1}{n}} (q_{c\sigma} q'_{c\tau} \ldots .) \end{aligned}\right\}. \quad (92)$$

But again, because $b\beta = n = yv$, and y is not a multiple of b, v is a multiple of b. Therefore $v\beta$ is a multiple of $b\beta$ or n. Therefore $F_{c\beta}^{\frac{-v\beta}{n}}$ is a rational