

To verify this result,

$$\begin{aligned}\frac{1}{81}(1 + \sqrt{2})^5 &= 1.012496, \\ \frac{1}{81}(1 - \sqrt{2})^5 &= -.000150535, \\ 197 + 139\sqrt{2} &= 393.57568, \\ (197 + 139\sqrt{2})^3 &= 154901.8, \\ 197 - 139\sqrt{2} &= .424315, \\ (197 - 139\sqrt{2})^3 &= .1800434, \\ \sqrt{\left\{(197 + 139\sqrt{2})^3 - \frac{1}{81}(1 + \sqrt{2})^5\right\}} &= 393.57436, \\ \sqrt{\left\{(197 - 139\sqrt{2})^3 - \frac{1}{81}(1 - \sqrt{2})^5\right\}} &= .180194.\end{aligned}$$

Therefore

$$\begin{aligned}u_1 &= 5.888, \\ u_4 &= .412, \\ u_2 &= 1.502, \\ u_3 &= -.276, \\ \therefore x &= 7.526.\end{aligned}$$

MODIFICATION OF THE METHOD TO MEET SPECIAL CASES.

First special case: When p_2 and p_3 are both zero.

§23. When p_2 and p_3 are both zero, a modification of the general method is rendered necessary by the circumstance that the equations (11) and (16), from which y and t are to be found, are then virtually one, and so are insufficient to give us the values of y and t . In fact, they become

$$\begin{aligned}& \left. \begin{aligned}t\left(50y^3 - \frac{2}{5}p_4y\right) - p_5y &= 0 \\ \text{and } \left\{t\left(50y^3 - \frac{2}{5}p_4y\right) - p_5y\right\}^2 &= 0\end{aligned} \right\} \quad (39)\end{aligned}$$

§24. In an article which appeared in No. 2, Vol. VII of this *Journal*, the present writer showed that, when p_2 and p_3 are both zero, p_4 and p_5 have the forms

$$\begin{aligned}p_4 &= \frac{5n^4(3-m)}{16+m^2} \\ p_5 &= \frac{n^5(22+m)}{16+m^2}\end{aligned} \quad (40)$$