Fractions.

In our schools too much time is given to arithmetic,—and too little is known about the subject when we are done. The reason is, to a large extent, that we do not base our work on sense-perception; we deal with symbols that symbolize nothing. We manipulate *figures* with little regard to *numbers*. With many a man, what he knows of figures, he has learned at school; what he knows of numbers, he has learned on the play-ground, and in contact with things and affairs,—in *spite* of the schools.

Many pupils find fractions the most difficult subject in arithmetic. There is no good reason for this; if the conceptions of fractions are gained from sense-perception, they should present no great difficulty to one who has properly learned integers. In fact, he will find little or nothing in fractions that is really new.

I propose to illustrate these statements by a few brief examples.

Fractions are not "broken numbers;" there is no such thing.

A number is one, or a collection of ones of the same kind.

A fractional number is a number of relative ones, smaller than some primary unit to which their relation is expressed. For instance, four-fifths of an apple is *four* with all the properties and characteristics of any other four; but it is four whose relation to the primary unit—*one apple*—is such that it takes five of the fractional ones to make the primary one. This fact, and the similar fact in regard to any fraction, a child can be made to see clearly by illustrations with objects.

I have found silver coins good objects to use in this work; children are interested in them, and they are not perplexed with any thought of some supposed troublesome "division." Show the child 3 quarter-dollars; ask, "How many such would make a dollar?" Let him write the figure 3 and see that it answers the question, How many? Now teach him to write 4, the number of units of this kind which it takes to make a whole, under the 3, separating them by a short horizontal line. The four answers the question, "What kind?"

Adhere strictly to the view of a fraction thus developed. The child will thus come to understand that the *numerator* of a fraction expresses the number; and that the denominator is simply a "modifier" of the numerator in showing the relation of these ones to a primary one.

Now, how easy to show that increasing the denominator diminishes the value of the fraction, while diminishing the denominator increases the value of the fraction. By the use of objects, this can be made perfectly clear to any bright seven-year-old.

So, it will be easy to show that the value of the fraction is unchanged when both numerator and denominator are multiplied or divided by the same number.

Take $\frac{3}{4}$ of some object. A cake I used recently. Let the child write the fraction $\frac{3}{4}$ on paper, or on the slate. Break each of the three pieces in halves. Let him write the new number of pieces, 6; lead him to see that it will take twice as many of these pieces to make a whole; then to write the new denominator, 8. He knows that he has had the same quantity of cake all the time.

Take $\frac{8}{12}$ of some object,—a paper circle, for instance. Fasten the pieces together by 4's; he now has 2 pieces. Get him to see that 3 such pieces would make a whole; he can now write the new fraction, $\frac{2}{3}$. When, through exercises of this kind judiciously presented, he grasps the truth, he knows all about "reducing fractions to their lowest terms." If the terms of the fraction are large, it is well to resolve them into their prime factors. The pupil should have learned to do this readily, as in the following example:

$$\frac{2835}{3402}=\frac{5}{6}$$

$$2835=3^2\times3^2\times5\times7$$
315
35

$$3402=2\times3^2\times3^2\times3\times7$$
1701
189
21

Resolving, as above, and rejecting common factors, there remain 5 in the numerator and 2 and 3 in the denominator; hence the result is $\frac{5}{6}$.

To change a mixed number to an improper fraction, take the example $5\frac{2}{3}$. Here are two unlike numbers 5 and 2 to be added together. We can make them alike by changing the 5 to thirds. As there are $\frac{3}{3}$ in 1, in 5 there will be 5 times $\frac{3}{3}$ or $\frac{15}{3}$. Now, the result is $\frac{15}{3}+\frac{2}{3}=\frac{17}{3}$. Let the pupil do this by taking 5 objects and cutting them up into thirds, etc. If the work is done without objects, be sure that he says "5 times $\frac{3}{3}$;" do not allow the multiplication of 5 by 3.

The opposite process is quite as obvious. Take $\frac{12}{5}$ of some object, and find how many wholes it equals. Of course, as $\frac{5}{5}$ make a whole, there will be as many wholes as there are times $\frac{5}{5}$ in $\frac{12}{5}$.

But suppose we are to change a mixed number to a fraction having a given denominator, as $\frac{8}{3}$ = how many sevenths? In order that the parts may be made into sevenths, we must cut each of the present 8 parts into 7 equal pieces; this will give us $\frac{56}{21}$. Every 3 of these 21sts will make one seventh; hence, we shall have

$$\frac{18\frac{2}{3}}{7}$$

This is the process if we use objects. If we use figures only, we may say, "I have $\frac{8}{3}$; but I want an equivalent fraction whose denominator shall be 7. Multiplying the present denominator by $2\frac{1}{3}$, we have 7, and multiplying the numerator by the same number, we have

$$\frac{18\frac{2}{3}}{7}$$ as before.

—*Public School Journal.*