5. Prove that if the straight line which bisects the vertical angle of a triangle, also bisects the base, the triangle is isosceles.

6. Simplify:—

$$2\left\{3a-2(b-3c)\right\}-4\left\{-2b-3(2c-a)\right\}+\left\{c-3(-b-2a)\right\}$$

7. Divide $1+2x-19x^6+16x^7$ by $1+4x+7x^2+10x^3+13x^4+16x^5$.

8. Divide $x^3-2ax^2+(a^2-ab-b^2)x+ab(a+b)$ by $x-(a+b)$.

9. Resolve into elementary factors:—
$5x(x^2-y^2)+3x(x-y)^2-12x(x-y)y$, and $12x^2-31xy+20y^2$.

10. Solve the following equations:—
(a) $a(x-b)=b(a-x)-(a+b)x$.
(b) $\frac{1}{12}x-\frac{1}{8}(8-x)-\frac{1}{4}(5+x)+\frac{11}{4}=0$.
(c) A man and his wife could drink a barrel of beer in 15 days. After drinking together for 6 days, the woman alone drank the remainder in 30 days. In what time would either alone drink it?

EDUCATION DEPARTMENT ONTARIO, JULY EXAMINATIONS, 1884.

ALGEBRA.

SECOND CLASS TEACHERS.

Examiner—J. C. Glashan.

1. Show that $(x+y+z)^5-x^5-(y+z)^5$ is exactly divisible by $x(y+z)(x+y+z)$.

2. Write down the factors of
$x^3-(a+b+c)x^2+(ab+bc+ca)x-abc$, and apply your result to obtain the factors of
(a) $(a+b+c)(ab+bc+ca)-abc$;
(b) $2(a+b+c)^3+(a+b+c)\{a(b+c)+b(c+a)+c(a+b)\}-(a+b)(b+c)(c+a)$.

3. If $3x=2(q+r)-p$, $3y=2(r+p)-q$ and $3z=2(p+q)-r$ then shall $x^2+y^2+z^2=p^2+q^2+r^2$ and $xy+yz+zx=pq+qr+rp$.

4. If $\frac{ax}{b-c}=\frac{by}{c-a}=\frac{cz}{a-b}$ then shall $ax+by+cz=0$, and $a^2x+b^2y+c^2z=0$.

5. If $a=-\frac{1}{2}(1+\sqrt{-3})$ then shall $\frac{1}{a}=-\frac{1}{2}(1+\sqrt{-3})$, and $a^3+\frac{1}{a^3}=2$.

6. Solve—
(a) $\frac{4}{x-1}-\frac{1}{x-4}=\frac{9}{x-2}-\frac{6}{x-3}$;
(b)
$$\left(\frac{x}{a}+\frac{x}{b}-1\right)\left(\frac{x}{a}-\frac{x}{b}+1\right)+\left(\frac{x}{b}+\frac{x}{c}-1\right)\left(\frac{x}{b}-\frac{x}{c}+1\right)+\left(\frac{x}{c}+\frac{x}{a}-1\right)\left(\frac{x}{c}-\frac{x}{a}+1\right)=1;$$
(c) $(2x^2-2x+1)^2=(x-2)^2$.

7. Solve the simultaneous equations—
$x^2-y^2=xy+1$, $x^2+y^2=2(xy+2)$.

8. A boy spends his money in oranges. Had he got five more for his money they would have averaged a cent each less, but had he got three less they would have averaged a cent each more. How much did he spend?

9. Find a number such that if it be divided into any two parts whatsoever, the square of one of these parts added to the other part will be equal to the square of the latter added to the former.

SOLUTIONS.—1. Put $x=0$, and we have $(y+z)^5-(y+z)^5=0$.
Put $y+z=0$, and we have $x^5-x^5=0$.
Put $x+y+z=0$, *i. e.*, $y+z=-x$,
and we get $0+(y+z)^5-(y+z)^5=0$.
∴ x, $y+z$, and $x+y+z$ are factors.—SEE TEACHERS' HANDBOOK, p. 48.

2. $(x-a)(x-b)(x-b)$.—SEE HANDBOOK, p. 10, 4, Bk.
(a) Comparing this with the given expression we see that x corresponds to $a+b+c$, and that the first two terms vanish when we substitute $a+b+c$ for x in the first expression. Hence $(a+b+c-a)$, $(a+b+c)-b$, $(a+b+c)-c$, or $(b+c)(c+a)(a+b)$ are the factors.

(b) For $(a+b)(b+c)(c+a)$ substitute the expression in (a), and we get
$2(a+b+c)^3+2(a+b+c)(ab+bc+ca)-(a+b+c)(ab+bc+ca)+abc$,
or, $(a+b+c)^3+(a+b+c)(a+b+c)^2+(a+b+c)(ab+bc)+ca)+abc$.
Comparing this with $x^3+(a+b+c)x^2+(ab+bc+ca)x+abc=(x+a)(x+b)(x+c)$, we see that the factors must be
$(a+b+c)+a$, $(a+b+c)+b$, $(a+b+c)+c$
or, $(2a+b+c)(a+2b+c)(a+b+2c)$.

3. Adding we have $x+y+z=p+q+r$. (A)
Squaring (1), (2) and (3), and adding, we get
$x^2+y^2+z^2=p^2+q^2+r^2$ (B). Squaring (A) and comparing with (B) we have $xy+yz+zx=pq+qr+rp$.

4. $\frac{ax}{b-c}=\frac{by}{c-a}=\frac{cz}{a-b}=\frac{ax+by+cz}{0}$.
∴ $ax+by+cz=0$.—SEE HANDBOOK, p. 123.
Again $\frac{a^2x}{a(b-c)}=\frac{b^2y}{b(c-a)}=\frac{c^2z}{c(a-b)}=\frac{a^2x+b^2y+c^2z}{0}$.
∴ $a^2x+b^2y+c^2z=\{a^2x\div a(b-c)\}\times 0=0$.

5. Invert and rationalise, and $\frac{1}{a}=-\frac{1}{2}(1+\sqrt{-3})$
Adding, we have $a+\frac{1}{a}=-1$, cube by formula 6,
∴ $a^3+\frac{1}{a^3}+3(a)\left(\frac{1}{a}\right)(-1)=(-1)^3=-1$; ∴ $a^3+\frac{1}{a^3}=2$.
—SEE HANDBOOK, p. 11.

6. (a) Add each side separately, and
$(3x-15)\div(x^2-5x+4)=(3x-15)\div(x^2-5x+6)$,
∴ $3x-15=0$, or $x=5$, one solution.
Also $x^2-5x+4=x^2-5x+6$,
or $x-5+\frac{4}{x}=x-5+\frac{6}{x}$, *i. e.* $\frac{4}{x}=\frac{6}{x}$.
Now this can only be true when x is endlessly increased, so that $\frac{4}{x}=0=\frac{6}{x}$. Hence $x=\infty$ is the other solution.

(b) 1st product $=\left\{x\left(\frac{1}{a}+\frac{1}{b}\right)-1\right\}\left\{x\left(\frac{1}{a}+\frac{1}{b}\right)-1\right\}$
$=x^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)+\frac{2x}{7}-1$, Hence by symmetry,
2nd " $=x^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)+\frac{2x}{c}-1$
3rd " $=x^2\left(\frac{1}{c^2}-\frac{1}{a^2}\right)+\frac{2x}{a}-1$
Sum $=2x\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-3=1$
∴ $x=2\div\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=2abc\div(ab+bc+ca)$.
—See HANDBOOK, Chap. II., *Symmetry.*

(c) $2x^2-2x+1=x-2$, or $-x+2$.
∴ $2x^2-3x+3=0$; or $2x^2-x-1=0$ ∴ $x=$&c.

7. From (2) $x-y=\pm2$ ∴ $x=y\pm2$, ∴ from (1)
$(y\pm2)^2-y^2=(y\pm2)y+1$;
or $y^2\pm4y+4-y^2=y^2\pm2y+1$, *i. e.* $y^2\mp2y-3=0$
$y=\pm3,\pm1$, $x=\pm5$, ±3.

8. Let $x=$ No. and $y=$ price each ∴ $xy=$ amount spent, and $(x+5)(y-1)=xy=(x-3)(y+1)$, ∴ $x=4$, $y=15$, $xy=60$c.

9. Let $x=$ No. and a and b the parts.
∴ $x=a+b$; and $a^2+b=b^2+a$, *i. e.* $a^2-b^2=a-b$;
or $(a+b)(a-b)=a-b$ ∴ $a-b=0$, or $a=b$ one solution.
Also $a+b=1=x$, another solution. The first solution requires the parts to be equal and does not apply to the problem.—ANS. 1.