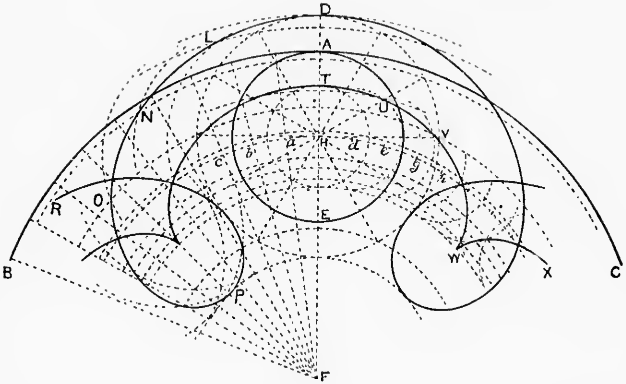

PROBLEM 82—TO DRAW THE HYPOTROCHOIDAL CURVES.

These are described when the generating circle rolls within the directing circle. The construction is similar to the foregoing. To find the curve traced by the point **D** when the circle **A E** rolls within the circumference **B A C**: Divide the circumference of the circle **A E** into any number of equal parts, and on **B A C** mark off on either side of **A** parts equal to these. Join the points so found to the centre **F** cutting the arc described through the centre **H** of the rolling circle in **a, b, c, d, e, g, etc.**

Through the points in the circumference of the circle having a radius **H D**, describe arcs from the centre **F** and from the centres **a, b, c, etc.**, describe arcs with radius equal to **H D** meeting the former in **L, N, O, etc.** This curve, the superior hypotrochoid, will form loops as **N O P R.**

The inferior hypotrochoidal curve, traced by the point **T** of which **T U V W X** is a part,, is described in a similar man er.

INVOLUTES IN GENERAL, AS INVOLUTES OF AN ELLIPSE, CYCLOID, ETC.

The path of a point in a flexible inextensible thread made to unwind itself tangentially from a curve, in the plane of the curve, is an **involute.**

The curve from which the line is unwound is called the **evolute** of the involute curve. The involute curve being given, its evolute may, by a converse operation, be determined. To form an involute, tangents are drawn from the evolute, and the points of tangency are the instantaneous centres of curvature.

If from the involute normals be drawn and the centres of curvature obtained, the locus of these centres will be the evolute. (See figure and also Problem 76.)

To obtain the centres of curvature when the character of the curve is not known, it is necessary to assume three points near to each other, and obtain the centre of the circle passing through them. (Prob. 36.)

As any curve may be considered an evolute from which to obtain an involute, so also it may be considered an involute from which the corresponding evolute may be obtained.

CYCLOIDS IN GENERAL.

A Cycloid is described by any point in a curve when the curve rolls on any other curve. As for example, an ellipse rolling upon an ellipse.

As an exercise, describe the cycloid generated by a point in an ellipse having axes of 3″ and 1″ when the director is a straight line.