ARTS DEPARTMENT.

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Our correspondents will please bear in mind, that the arranging of the matter for the printer is greatly facilitated when they kindly write out their contributions, intended for insertion, on one side of the paper ONLY, or so that each distinct answer or subject may admit of an easy separation from other matter without the necessity of having it re-written.

Solutions have been received from Messrs. Frisby, Cox, McMinn, Ellis, Boultbee, and MacMurchy, which will be made use of in subsequent numbers of the MONTHLY.

SOLUTION

BY PROPOSER, J. L. COX., B.A., MATH. MASTER, COLLINGWOOD C. I.

45. In a given triangle to inscribe a triangle similar to a given triangle.

Let ABC be the given triangle in which the triangle is to be insciibed. In AB take any point D and draw any line DF to the adjacent side; and at the points D and F make the angles FDE, DFE, equal to two of the angles of the given triangle to which the inscribed one is to be similar; therefore, the angle E will be equal to the third angle. Join AE, and produce it to G; and from G draw GH, GI, respectively, parallel to ED, EF; join HI. HIG is the triangle required.

Since DE and EF are respectively parallel to HG, GI, the angle DEF is equal to HGI.

Also $DE : HG :: AE : AG :: EF : GI$, whence (EUC. VI. 6) the triangles HGI, DEF, are similar, and therefore HGI is similar to the given triangle.

60. Prove (by the method of Indeterminate Coefficients) that the sum of the products of the first n natural numbers, taken two and two together, is

$$\frac{(n-1)\,n\,(n+1)\,(3n+2)}{24}$$

Let Σ_n be the required sum

Assume $\Sigma_n = A_0 + A_1 n + A_2 n^2 \ldots.$

$\because \Sigma_{n+1} = A_0 + A_1(n+1) + A_2(n+1)^2 \ldots$

$\therefore (n+1)(1+2+3\ldots.n) = \Sigma_{n+1} - \Sigma_n = A_1 + A_2(2n+1) + A_3(3n^2+3n+1) + \ldots.$

or, $\dfrac{n^3+2n^2+n}{2} = A_1 + A_2(2n+1) + A_3(3n^2+3n+1) + A_4(4n^3+6n^2+4n+1) +$

Equating co-efficients of n in these identical equations.

$$\left.\begin{array}{r} A_1 + A_2 + A_3 + A_4 = 0 \\ 2A_2 + 3A_3 + 4A_4 = \frac{1}{2} \\ 3A_3 + 6A_4 = 1 \\ 4A_4 = \frac{1}{2} \end{array}\right\} \therefore \begin{array}{l} A_1 = -\frac{1}{12}. \\ A_2 = \frac{1}{8}. \\ A_3 = \frac{1}{12}. \\ A_4 = \frac{1}{8}. \end{array}$$

$$\therefore \Sigma_n = A_0 - \frac{2 + 3n - 2n^2 - 3n^3}{24} n.$$

Put $n=2$, and therefore $A_0 = 0$, and

$$\Sigma_n = \frac{(n-1)\,n\,(n+1)\,(3n+2)}{24}.$$

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104. $(x^2 + xy + y^2)^2 - (x^2 + xy + y^2)(x^2 + y^2) + x^2 y^2$

$= (x^2+xy+y^2)(x^2+xy+y^2-x^2-y^2) + x^2y^2$

$= (x^2+xy+y^2)(xy) + x^2y^2$

$= xy(x+y)^2$

—FRANK BOULTBEE, *Univ. Coll.*