(iii.) $\begin{cases} x^3+y^3+z^3=3xyz, \\ y+z=x-a, \\ yz=bx. \end{cases}$

8. If u and x are connected by the quadratic equation

$$\frac{au^2+bu+c}{du^2+eu+f}=x,$$

show that the values of x corresponding to which u has coincident values are given by

$(e^2-4df)x^2+2(2af+2dc-be)x+(b^2-4ac)=0.$

If m, n are the roots of the equation

$$x^2-px+q=0,$$

find the equation whose roots are $\frac{1}{m}$ and $\frac{1}{n}$.

9. A grocer sold 60 lbs. of coffee and 80 lbs. of sugar for \$25, but he sold 24 lbs. more of sugar for \$8 than he did of coffee for \$10. What was the price of a pound of each?

EDUCATION DEPARTMENT, ONTARIO.

JULY EXAMINATIONS, 1882.

Intermediate.

ALGEBRA.

1. Form an expression symmetrical with respect to x, y, z, u, similar to $x^3+y^3+z^3-3xyz$, and write down the quotient on dividing it by $x+y+z+u$.

Required expression is

$x^3+y^3+z^3+u^3-3xyz-3xyu-3xzu-3yzu,$

and quotient is

$x^2+y^2+z^2+u^2-yz-zx-xy-xu-yu-zu.$

2. Factor $ax^3-(a+b)(x-y)xy-by^3$.

Deduce, or find by other means, the factors of

$(a+b)^3(x+y)-(x+2y+z)(a-c)(a+b)(b+c)-(b+c)^3(y+z).$

Obtain four different relations between the quantities a, b, c, d, for any one of which the expression $4(ad-bc)^2-(a^2+d^2-b^2-c^2)^2$ will vanish.

$$\begin{aligned} ax^3&-(a+b)(x-y)xy-by^3 \\ &=a\left\{x^3-(x-y)xy\right\}-b\left\{y^3+xy(x-y)\right\} \\ &=(ax-by)(x^2-xy+y^2). \end{aligned}$$

Writing in last result for a, $x+y$, for b, $y+z$, and so on, factors required are

$$\left\{(x+y)(a+b)-(y+z)(b+c)\right\}\left\{a^2+b^2+c^2+bc-ca+ab\right\}.$$

Factoring,

$$\left\{(b-c)^2-(a-d)^2\right\}\left\{(a+d)^2-(b+c)^2\right\}=0.$$

Equating each factor to zero we find

$$\left.\begin{aligned} a+b+c+d&=0 \\ a-b-c+d&=0 \\ -a+b-c+d&=0 \\ a+b-c-d&=0 \end{aligned}\right\}.$$

3. Find the lowest common measure, not being a fraction, of the quantities

$$\frac{x^2+5x+6}{x+4} \text{ and } \frac{x^2+7x+12}{x+5}.$$

$$(x+2)(x+3)(x+4).$$

4. Reduce to lowest terms the following fractions:

(1) $\dfrac{6x^5-5x^4-1}{x^5-x^4-x+1}$;

(2) $\dfrac{(a-b)(b-c)(c-a)}{(a-b)^3+(b-c)^3+(c-a)^3}$.

(1) $\dfrac{6x^4+(x+1)(x^2+1)}{x^4-1}$; (2) $\dfrac{1}{3}$.

5. (1) If $y+z+u=a$, $z+u+x=b$, $u+x+y=c$, $x+y+z=d$, then

$$\frac{1}{1+\frac{a}{x}}+\frac{1}{1+\frac{b}{y}}+\frac{1}{1+\frac{c}{z}}+\frac{1}{1+\frac{d}{u}}=1.$$

(2) If $ax=b+c$, $by=c+a$, $cz=a+b$, then

$$\frac{1}{1+x}+\frac{1}{1+y}+\frac{1}{1+z}=1.$$

(1) $\dfrac{1}{1+\frac{a}{x}}=\dfrac{x}{x+y+z+u}$, etc.; $\therefore$ sum $=1$.

(2) $\dfrac{1}{1+x}=\dfrac{1}{1+\frac{b+c}{a}}=\dfrac{a}{a+b+c}$, etc.;

$\therefore$ sum $=1$.

6. Solve the equation $ax^2+bx+c=0$.

What value of x will satisfy the equation

$$\frac{b-c}{x+a}+\frac{c-a}{x+b}+\frac{a-b}{x+c}=0.$$

Bookwork; both values of x are infinity.

7. Solve the equations

(1) $\dfrac{7x}{3}-\left\{\dfrac{1}{2}-\left(\dfrac{x}{3}-\dfrac{x-1}{2}\right)\right\}=\dfrac{4x-2}{5}.$