two roots. Must it always have two?

What are the roots of $x + \frac{1}{x} = 2$?

11. Find the conditions that the roots of the equation $ax^2 + bx + c = 0$ may be positive integers.

12. Find the arithmetic, geometric and harmonic means between two given quantities.

For examples, between 2 and $\frac{1}{2}$, and between -2 and $-\frac{1}{2}$.

13. If A G H be the arithmetic, geometric and harmonic means between a, b, then will

$$\frac{H}{A} = 1 + \frac{(H-a)(H-b)}{G^2}$$

14. Having given the first and last of n quantities in arithmetic progression, find their sum.

There being 49 terms, of which the first is 200 and the last -100, find their sum and the middle term.

15. Given the last term l, the sum s and the common difference d construct the series.

Shew that there will be two distinct series, provided the following conditions hold:

(1). d and s have the same sign;

(2). $(2l + d)^2 - 8ds$ is finite and a complete square (z^2 suppose);

(3). $2l + z$ and $2l - z$ each an odd multiple of d. And when this is the case, if a, b be the first terms in these two series, then will $a + b = d$.

16. In a geometric series, given the first term, the common ratio and the number of terms, find the sum; and when the common ratio is a proper fraction find the limit to which the sum approaches as the series is continued indefinitely.

Find the sum to n terms, of

$$\frac{2}{3} - 1 + \frac{3}{2} - \frac{9}{4} + \ldots.$$

and *ad infinitum* of

$$\frac{3}{2} - 1 + \frac{2}{3} - \frac{4}{9} + \ldots.$$

17. If the common ratio r in a geometric series, whose first term is 1, differ but little from unity, an approximation to the sum of n terms will be furnished by

$$s = n + \frac{n\,\overline{n-1}}{2}\,\overline{r-1}$$

and a still closer approximation will be

$$s + \frac{n-2}{3}\,\overline{s-n}\,.\,\overline{r-1}$$

18. Solve the equations:

(1). $(x^2 - 2)(x + 1) = (x - 2)(x^2 + 1)$

(2). $(ax + b)(px + q) = (bx + a)(qx + p)$

(3). $x^2 - 11x\frac{112}{121} = 11(x + 11) + \frac{18}{121}$

(4). $xy(x^2 + y^2) = a$

$$\frac{x}{y} - \frac{y}{x} = b$$

19. Find the time between successive transits of the minute hand over the hour hand of a common clock.

20. The opposite sides of a rectangle are each increased by a length a, and the other two sides each diminished by b, and the area is found to be unaltered; but if these changes in the sides had been respectively c, d, the area would have been diminished by e. Find the sides and examine the nature of the problem when $ad = bc$ and $bc + e = cd$.

SOLUTIONS TO PROBLEMS FROM CORRESPONDENTS.

NOTE.—*When a solution to a problem sent does not appear, the sender will confer a favor by repeating the problem.*

1. What must be the least number of soldiers in a regiment to admit of its being drawn up 2, 3, 4, 5 or 6 deep and also admit of its being drawn up in a solid square?

To admit of being drawn up 2, 3, 4, 5 or 6 deep the number must be a multiple of each of these numbers, and ∴ must be at least 60. Also, since every square number contains each of its simple factors an even number of times, therefore, this number must contain 2, 3, 5 each an even number of times, and therefore must contain 4, 9 and 25, and hence must be at least 900.