7. 19(volume of gold)+2·5(do. of quartz) =7(volume of gold+volume of quartz).

12(volume of gold)=$4\frac{1}{2}$(volume of quartz).

∴ volume of gold=$\frac{3}{11}$ of whole volume.

c. in.
$\frac{3}{11}\times\frac{1728}{7000}\times 19$=wt. of gold in 1 oz. mixture.

8. At the time the goods are sold the man has really paid $\$\frac{100}{102}\times\frac{520}{1}$ for them, which is present worth of \$520 due 3 months hence at 8 %. He sells them for $\frac{467}{400}\times\frac{100}{102}\times\frac{520}{1}$. Now, for what term must this sum be put out at interest to equal \$677.70?

Answer.

$$\frac{\$677.70-\$\frac{467}{400}\times\frac{100}{102}\times\frac{520}{1}}{\frac{467}{400}\times\frac{100}{102}\times\frac{\$520}{1}}=\cdot 13+\text{ years.}$$

9. $\frac{\text{percentage}}{100}\times\frac{6000}{1}=60\times\text{percentage}=$ 1st charge. Now, for the 2nd commission he gets $\frac{1}{25}$ of 2nd price, which equals $\frac{1}{25}$ of (6000 − 60×percentage), since $\frac{1}{25}\times\frac{25}{25}=\frac{1}{25}\times 1$ $60\times\text{percentage}+\frac{1}{25}(600-60\times\text{percentage})=375$. Whence percentage

$=\frac{\cancel{3750}^{5}}{\cancel{25}}\times\frac{\cancel{25}}{\cancel{25}\times\cancel{60}}=\frac{5}{2}=2\frac{1}{2}$. *Ans.* $2\frac{1}{2}$ %.

10. (i.) $\frac{112\frac{1}{2}}{360}\times\frac{22}{7}\times\frac{6400}{1}=6285\frac{5}{7}$ sq. yds.;

$\frac{112\frac{1}{2}}{360}\times\frac{22}{7}\times\frac{160}{1}=157\frac{1}{7}$ yds.

(ii.) A rectangle can easily be formed, by dropping a perpendicular from one of the angles of the quadrilateral to the opposite side, whose sides are 3 and 4; ∴ area = 12. Area of remaining △ (which is right-angled) = 6. Whole area = 18.

ALGEBRA.—SOLUTIONS.

First Class.

1. Writing p, q, s for $a-b$, $b-c$, $c-a$ in the expression and transposing, we have $2(p^7+q^7+s^7)-7pqs(p^4+q^4+s^4)=0$. Now $p+q+s$ is a factor of the expression on the left hand side of this $=n$, and $p+q+s=a-b+b-c+c-a=0$. ∴ the $=n$ is an identity.

2. *Ans.* $2+\sqrt{-1}+2-\sqrt{-1}=4$.

3.

(1) Factoring, we obtain

$(2x^2-5x+2)(x^2+3x+1)=0$.

$(x-2)(2x-1)(x^2+3x+1)=0$.

Solve by equating factors to zero. $x=2$. $\frac{1}{2}$, or $\frac{-3\pm\sqrt{5}}{2}$.

(2) $x^2+y^2+z^2+2xy+2xz+2yz=a^2+2b^2$

$x+y+z=\pm\sqrt{a^2+2b^2}$

$x+y+z=c$.

∴ $z=\frac{1}{2}(\pm\sqrt{a^2+2b^2}-c)$.

Write down the values of x and y from the value of z in circular order.

(3) Factoring $\sqrt{x+4}\,(\sqrt{x+1}+\sqrt{x-1})=(\sqrt{x+4})^2$. $\sqrt{x+4}=0$ $x=-4$.

Squaring both sides after dividing by $\sqrt{x+4}$, collecting coefficients and dividing through by coefficients of x, $x=\frac{-4\pm 2\sqrt{19}}{3}$.

4. See Todhunter's Larger Algebra, Art. 634.

5. $\frac{x^2-xy+y^2}{xy}=\frac{a^3+b^3}{ab(a+b)}=\frac{a^2-ab+b^2}{ab}=$

$\frac{x}{y}+\frac{y}{x}=\frac{a}{b}+\frac{b}{a}$.

$\frac{x}{y}-\left\{\frac{a}{b}+\frac{b}{a}\right\}+\frac{y}{x}=0=\frac{x^2}{y^2}-\left\{\frac{a}{b}+\frac{b}{a}\right\}\frac{x}{y}+1$

$=\left\{\frac{x}{y}-\frac{a}{b}\right\}\left\{\frac{x}{y}-\frac{b}{a}\right\}=\frac{y}{a}\left\{\frac{x}{y}-\frac{a}{b}\right\}\frac{y}{b}$

$\left\{\frac{x}{y}-\frac{b}{a}\right\}=0$

$=\left\{\frac{x}{a}-\frac{y}{b}\right\}\left\{\frac{x}{b}-\frac{y}{a}\right\}=0$.

6. $n=6$ or -8.

When $n=6$, the series referred to is that, the first term of which is 6, the second 10, the third 14, etc. When, in the series 6, 10, ... 26, $n=-8$, we are to begin at 26 and count backwards 8 terms. The progression will thus be 26, 22, 18, 14, 10, 6, 2, −2.

7. Let α, α, β be the roots of $x^3+0.x^2+px+q$,

then $\alpha+\alpha+\beta=0$

$\alpha^2+\alpha\beta+\alpha\beta=p$

$\alpha^2\beta=-q$

whence $2\alpha^3=q$, $-3\alpha^2=p$.

∴ $\left\{\frac{q}{2}\right\}^{\frac{1}{3}}=\left\{-\frac{p}{3}\right\}^{\frac{1}{2}}$.