Mathematics.

CORRESPONDENCE.

PROBLEMS, SOLUTIONS, ETC.

No. 63.—Sent by A.B., St. C. St., Montreal.

If $\frac{x^2+2x+1}{x^2-2x+3}=\frac{y^2+2y+1}{y^2-2y+3}$ show that each fraction is equal to $\frac{xy-1}{xy-3}$.

SOLUTION BY THE EDITOR: If $\frac{a}{b}=\frac{c}{d}$, then each $=\frac{a-c}{b-d}$ acd.

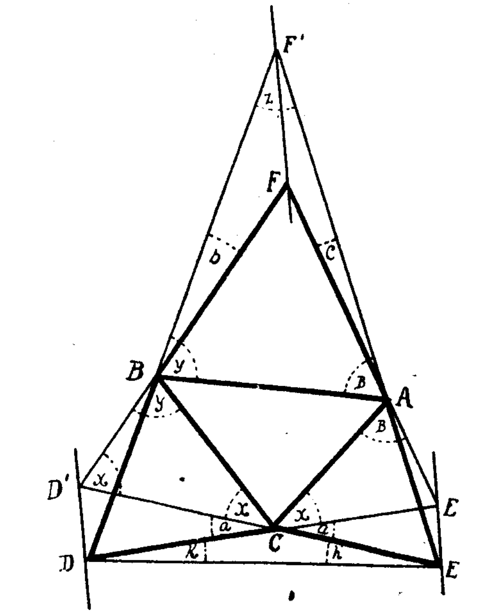
$$\therefore V=\frac{x^2-y^2+2(x-y)}{x^2-y^2-2(x-y)}=\frac{x+y+2}{x+y-2}=\frac{x^2+2x+1}{x^2-2x+3}$$

Apply $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$ and we have $\frac{x^2+2}{2x-1}=\frac{x+y}{2}$, whence $x+y=2\ (xy-2)$. Substitute this and $V=\frac{x+y+2}{x+y-2}=\frac{2xy-2}{2xy-6}=\frac{xy-1}{xy-3}$ Q.E.D.

No. 64. By L. M. Stevens, *Westerly, R.I.*—Let ACB be a triangle, and AFB, BDC, and CEA *equilateral* triangles on its sides, all *turned outwards*. Let FB and EC, produced, meet D', DC, and FA at E', EA and DB at F'. Show that the lines through DD', EE', and FF' are parallel.

[This property was recently discovered by Prof. Morley, of Haverford College, by means of complex variables, and it is sent to the JOURNAL by his permission. A geometric demonstration is desired.—ED.]

SOLUTION BY THE PROPOSER.



From the diagram,

$z+b+c=z+h+k=\frac{1}{3}\pi \ldots (1).$

$\therefore b+c=h+k=a \ldots (2).$

Now, $a+a=y-b=B-c=\frac{1}{3}\pi \ldots (3)$;

Adding. $a+B+y+a-(b+c)=\pi \ldots (4)$;

or by (2), $a+B+y=\pi \ldots (5).$

But $a+x+y=\pi \ldots (6).$

$\therefore x=B$; $y=y$; and $z=a$.

That is, the triangles ACE', CBD', and AF'B are similar.

$\therefore$ CE' : CA = CB : CD', or CE' : CE = CD : CD'. Hence the triangles CEE' and CDD' are similar; and, consequently. $\angle$CEE' = $\angle$CDD'. That is, EE' and DD' are parallel. Similarly DD' and FF' are proved to be parallel.—*N.E. Journal of Education, Boston.*

No. 65.—Sent by W. H. FLETCHER, WOODSTOCK.

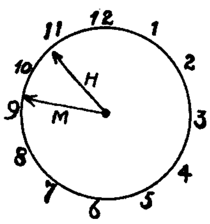
From the following Trial Balance construct a statement of Assets and Liabilities, and also of Profit and Loss, showing the surplus:

DEBIT ACCOUNTS.	MEMOS.		CREDIT ACCOUNTS.	MEMOS.	
	Int. Accrued			Interest and Dividend Accrued.	
Mortgages	$53,000	$1,500,000			
Stock Loans	218	10,000			
Government Bonds	427	15,000	Capital		$600,000
Office Premises		12,000	Dividend Due	$18,000	
Int. on Deposits and Debentures		10,000	Reserved Fund		260,000
Salaries and Expenses		15,000	Deposits	17,000	575,000
Dividends Paid		18,000	Debentures	2,000	103,000
Bank of Montreal		3,000	Int. Charged on Mortgages, etc.		38,000
Cash on Hand		245	Contingent Fund		7,245
		$1,583,245			$1,583,245

Commercial Specialists Examination, 1892.

The following solutions of Nos. 53, 56, 57, 58, 59 and 60, were sent in by MISS ETHEL BARKER, Jameson Avenue Collegiate Institute, Toronto.

NO. 53.—SOLUTION.



Suppose x = number minutes after 10.

Then hour hand has travelled $\frac{x}{12}$ spaces

$\therefore$ Distance from 12 $=10-\frac{x}{12}$

$\therefore$ Space between hour and minute hands $=10-\frac{x}{12}$

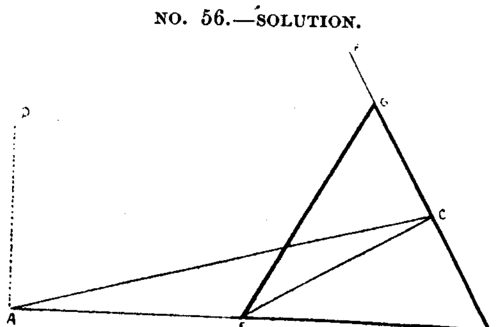
$\therefore 50+\frac{x}{12}-\left\{10-\frac{x}{12}\right\}=x$

$\therefore 480+2x=12x$

$10x=480$

x = 48 minutes after 10 o'clock.

NO. 56.—SOLUTION.



Let AB be the given sum of a side and the altitude of an equil. $\triangle$.

It is required to construct the $\triangle$.

At B make an angle of 60°, ABF, and at A erect a $\perp$ AD to AB.

Trisect $\angle$ DAB and bisect $\angle$ of 30° nearest AB by AC.

From C draw CE $\perp$ to BF, meeting AB in E.

From CF cut off GB = CB, join CE.

$\angle$CAB = 15°, $\angle$CBA = 60°

$\therefore$ $\angle$ ACB = 105°

$\angle$ BCE = 90°

$\therefore$ $\angle$ ECA = 15°

$\therefore$ EC = EA.

In $\triangle$'s EGB, ECB, EC = EC, CG = CB, $\angle$ECG = $\angle$ECB

$\therefore$ EG = EB

and $\angle$ EGC = $\angle$EBC = 60°

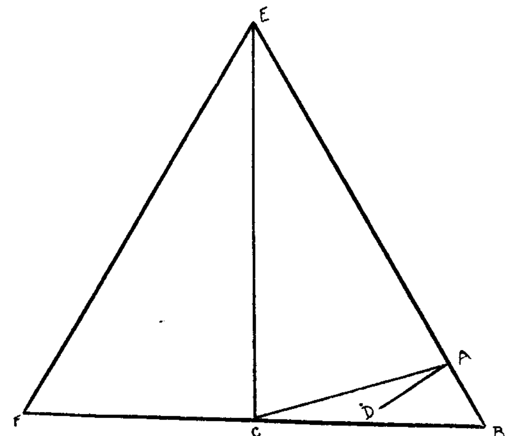
$\therefore$ $\triangle$ ECB is equiangular

$\therefore$ $\triangle$ EGB is equilateral

and EC + EB = AB

$\therefore$ $\triangle$ EBC is the required equil. $\triangle$.

NO. 57.—SOLUTION.



Let AB be the given difference of a side and the altitude of an equil. $\triangle$.

It is required to construct the $\triangle$.

On AB describe an equil. $\triangle$, and produce a side, BC, to C.

From A draw AD $\perp$ AB.

Trisect the complement of $\angle$ DAB and bisect the $\angle$ of 30° nearest AD.

From C draw CE $\perp$ CB, meeting AB in E.

From BC produced cut off CF = CB, join EF.

$\angle$ ABC = 60°, $\angle$ CAB = 105°

$\angle$ ACB = 15°

$\angle$ BCE = 90°

$\angle$ ACE = 75°

and $\angle$ CAE = 75°

$\therefore$ EC = EA

In $\triangle$'s ECB, ECF, EC = EC, CB = CF, $\angle$ ECB = $\angle$ ECF

$\therefore$ BE = FE

and $\angle$ CBE = $\angle$CFE

$\therefore$ $\triangle$ EFB is equiangular

$\therefore$ $\triangle$ EFB is equilateral:

and EB − EC = AB

$\therefore$ $\triangle$EFB is the required equil. $\triangle$

NO. 58,—SOLUTION.