

angles of the triangles DFA and EAF respectively be S and S_1 ; S is not greater than S_1 . For let s be the sum of the angles of the triangle ADE; then

$$S = F + FAE + EAD + D,$$

$$\text{and, } S_1 = F + FAE + AEF.$$

$$\begin{aligned} \therefore S_1 - S &= AEF - (EAD + D), \\ &= AEF + AED - (AED + EAD + D), \\ &= 2 - s; \end{aligned}$$

a right angle being taken as the unit of measure. But, by the Proposition, s is not greater than 2. Therefore S is not greater than S_1 .

COR. 2.—From B , a point within the triangle DAF , draw BC to a point C in AF ; and let S_2 be the sum of the angles of the triangle ABC . Then S_2 is not less than S . For, produce AB to E ; and join EC . Then, by Cor. 1, S_2 is not less than the sum of the angles of the triangle AEC ; which sum, again, is not less than S_1 , or the sum of the angles of the triangle AEF ; and S_1 is not less than S . Therefore S_2 is not less than S .

PROPOSITION II.

If any triangle CHE (Fig. 3) have S , the sum of its angles, equal to two right angles, every triangle has the sum of its angles equal to two right angles.

For, CE being a side which is not less than any other side of the triangle CHE , let fall HD perpendicular on CE . Then HD cannot fall without the base CE ; else (supposing it to fall beyond E) the angles CEH would be greater than a right angle: hence, because CE

is not less than CH , the angle CHE would be greater than a right angle: so that S would be greater than two right angles: which (Prop. I.) is impossible. Produce CD to F ; making $DF = CD$.

FIG 2

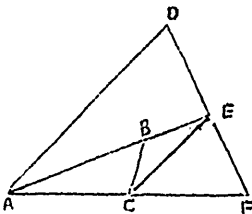


FIG 3

