

Prove that, if the difference between p and a perfect cube, N^3 , be less than one per cent. of either, p differs from $\frac{1}{3}N + \frac{1}{3}\frac{p}{N^2}$ by less than $\frac{N}{90000}$.

6. Find the number of combinations of n things taken r together.

Prove that, if each of m points in one straight line be joined to each of n in another by straight lines terminated by the points, then, excluding the given points, the lines will intersect $\frac{1}{2}mn(m-1)(n-1)$ times.

7. Define the tangent of an angle, and from the definition show that
 $\tan(180^\circ - A) = -\tan A$.

Prove directly from the definitions of the trigonometrical functions that

$$\frac{1 + \cos A}{\sin A} = \cot \frac{1}{2} A.$$

Find the general values of A from the equation $\tan A + \sec 2A = 1$.

8. Show *a priori* that when $\sin A$ is expressed in terms of $\sin 2A$, four values are to be expected generally.

If $\sin 2A = a$, what values of A give the following equation,

$$2 \sin A = -\sqrt{1+a} + \sqrt{1-a}?$$

Prove that, if $\sin 4A = a$, the four values of $\tan A$ are given by

$$\frac{1}{a} \left\{ (1+a)^{\frac{1}{2}} - 1 \right\} \left\{ 1 + (1-a)^{\frac{1}{2}} \right\}.$$

9. Prove that, if $A + B + C = 180^\circ$,

$$\begin{aligned} \sin^4 A + \sin^4 B + \sin^4 C \\ = \frac{3}{2} + 2 \cos A \cos B \cos C \\ + \frac{1}{2} \cos 2A \cos 2B \cos 2C, \end{aligned}$$

and that, if

$$\begin{aligned} \frac{\sin ra}{l} = \frac{\sin(r+1)a}{m} = \frac{\sin(r+2)a}{n}, \\ \frac{\cos ra}{2m^2 - l(l+n)} = \frac{\cos(r+1)a}{m(n-l)} = \frac{\cos(r+2)a}{n(l+n) - 2m^2} \end{aligned}$$

10. Prove that, if θ be the circular measure of an angle less than a right angle, $\frac{\sin \theta}{\theta}$ lies between 1 and $1 - \frac{1}{2}\theta^2$.

Find the value of $\sin 3''$ to ten places of decimals.

11. Find the area of a triangle in terms of one side and the adjacent angles.

If a triangle be cut out in paper and doubled over so that the crease passes through the centre of the circumscribed circle and one of the angles A , the area of the doubled portion is $\frac{1}{2} b^2 \sin^2 C \cos C \operatorname{cosec}(2C - B) \sec(C - B)$, C being $> B$.

12. It is observed that the altitude of the top of a mountain at each of the three angular points A, B, C of a plane horizontal triangle ABC is a : show that the height of the mountain is $\frac{1}{2} a \tan a \operatorname{cosec} A$.

Show that, if there be a small error n'' in the altitude at C , the true height is very nearly

$$\frac{1}{2} \frac{a \tan a}{\sin A} \left(1 + \frac{\cos C}{\sin A \sin B} \frac{\sin n''}{\sin 2a} \right).$$

PROBLEMS IN ARITHMETIC,

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1. Find the largest square number which is an exact divisor of 3780? *Ans.* 36.

2. The prices of seats at a lecture were 50 cts., 35 cts., and 25 cts. It is known that for every 3 seats sold at 35 cts. there were 4 at 25 cts., and for every 3 at 50 cts. there were 4 at 35 cts.; and the whole sum realized from the sale of seats was \$76.20. How many of each kind were sold?

Ans. 96 at 25 cts., 72 at 35 cts., and 54 at 50 cts.

3. At an examination $\frac{1}{4}$ of the candidates made 75 per cent. on the paper set, $\frac{1}{2}$ of the remainder made 60 per cent., $\frac{1}{3}$ of the remainder made 50 per cent., and half of the remainder failed; but the number who failed was greater by 9 than the number that made 75 per cent. How many were in the class? *Ans.* 48.

4. By what must $\frac{1}{2}$ of $\frac{3}{4}$ be diminished so that the result may be $\frac{1}{2}$ of that obtained by multiplying ($\frac{2}{3}$ of $\frac{3}{4} + \frac{1}{4}$) by ($\frac{3}{4} - \frac{1}{4}$ of $\frac{3}{4}$). *Ans.* $1\frac{1}{2}\frac{1}{2}$.

5. Which is greatest, .44, .44, or .44? Express the difference between each pair as a decimal, and determine in each case whether this decimal is finite, a mixed repetend, or a pure repetend. *Ans.* Last two are equal.

6. If 20 lbs. of green tea be mixed with