ARITHMETIC AND ALGEBRA.

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1. Enumerate the points of difference between algebra and arithmetic.

2. Given $a = 3$, $b = 4$, $c = -5$, find the value of $(3a + 4b + 5c)^2 + (4a + 3b + 12c)^2 - (5x + 5y + 13z)^2$

Given $a = 2$, $b = 1$, $c = 0$, find the value of

$$\left(\frac{b}{a} + \frac{c}{b} + \frac{a}{c}\right) \div \left(\frac{b}{a} + \frac{c}{b} + \frac{a}{c}\right)$$

3. Simplify

(1) $\left[a - 5b - \left\{a - (5c - 2c - b - 4b) + 2a - (a - 2b + c)\right\}\right]$

(2) $a - 2\left[b + 3\left\{a - 2(b - c) + 2b - 3(a - b + 2c)\right\}\right]$

4. Find the value of

(1) $$\frac{1 + 2x}{(3 - x)(1 + x)} + \frac{7}{(2 + x)(1 - 3x)} + \frac{x}{(1 + x)(2 + x)}$$

(2) $$\frac{1 - x}{1 + x} + \frac{1 - x - x^2}{1 + x + x^2} + \frac{1 - x - x^2 - x^3}{1 + x + x^2 + x^3}$$

(3) $$\frac{x^{3n}}{x^n - 1} - \frac{x^{2n}}{x^n + 1} - \frac{1}{x^n - 1} + \frac{1}{x^n + 1}$$

5. Describe Horner's method of division.

Divide according to Horner's method:

$4x^5 - 7x^4 + 25x^3 - 15x^2 + 8x + 10$ by $x^2 - x + 5$

$6a^4 - a^3b + 2a^2b^2 + 13ab^3 + 4b^4$ by $2a^2 - 3ab + 4b^2$

6. Prove the rule for finding the least common multiple of two quantities.

Find L. C. M. of $21x^2 - 26x + 8$ and $7x^3 - 4x^2 - 21x + 12$.

7. Solve

(1) $$(12 + x)^{\frac{1}{2}} = 2 + x^{\frac{1}{2}}$$

(2) $$\frac{x}{2} - \frac{\frac{1}{2}(2x - 3) - \frac{1}{4}(3x - 1)}{\frac{1}{2}(x - 1)} = \frac{3}{2}\left(\frac{x^2 + 2}{3x - 2}\right)$$

(3) $$\frac{66x + 1}{1{\cdot}5x + 1} \div \frac{4x + 5}{{\cdot}5x - 1} = 52$$

(4) $$\frac{\sqrt{a} - \sqrt{a - \sqrt{a^2 - ax}}}{\sqrt{a} + \sqrt{a - \sqrt{a^2 - ax}}} = b.$$

8. Find the fraction which, if 1 be added to its numerator, becomes $\frac{1}{3}$; but if one be added to its denominator becomes $\frac{1}{4}$.

Two persons A and B could finish a work in m days: they worked together n days when A was called off, and B finished it in p days. In what time could each do it?

9. Solve

1. $$\left(x + 22\right)^{\frac{1}{3}} - \left(x + 3\right)^{\frac{1}{3}} = 1$$

2. $$\frac{(a - x)^4 - (x - b)^4}{(a - x) - (x - b)} = \frac{(a - b)c}{(a - x)(x - b)}$$

3. $$\begin{cases} x + y + xy = 11 \\ x^2y + xy^2 = 30 \end{cases}$$

4. $$\begin{cases} x^{\frac{3}{4}} + y^{\frac{3}{5}} = 126 \\ x^{\frac{1}{4}} + y^{\frac{1}{5}} = 6 \end{cases}$$

10. The sum of two numbers added to the sum of their squares is 42, and their product is 15. Find the numbers.

If \$300 be laid out at simple interest for a certain number of years, it will amount to \$360. If the same be allowed to remain two years longer, and at a rate of interest one per cent. higher, it will amount to \$405. Find the rate and number of years.

11. Give the first term, common ratio, and number of terms in a geometrical progression, find the sum.

If a, b, c be in G. P. Prove

$$(a^2 + b^2 + c^2) > (a - b + c)^2$$