played by chance in the discovery of important facts, such as, doubtless, the above would be, in the history of a nation's mental development. It also serves to indicate the kind of stimulus that an appropriate study of empirical geometry should give to the *inventive* faculty of the child. Here, indeed, at once, we perceive a valuable educational parallelism such as we previously contemplated. We have, then, supposed the discovery of a certain relation, or law, between sides and area. The larger the number of cases tested, the stronger would be the belief in the universal applicability of the relation. But, however many be the tests, the law is still only an empirical statement; the two groups of numbers spoken of—the numbers giving respectively the area and the product of the sides—will never exhibit more than an approximate correspondence; the equality cannot, from the nature of the case, be absolutely exact. However valuable in future use the discovery may be, it is not a logically proved geometrical theorem, but a wide empirical induction. It ranks as a fact of experimental geometry, but forms no part of a scientific geometry. The relation might be discovered—and, indeed, appears to have been discovered—by one unversed in such abstractions as straight line, axiom, theorem, etc.

By way of sharp contrast, let the same problem of measuring a certain rectangular surface be now proposed to a man who grasps the spirit of a scientific geometry. He is aware that, from certain arbitrarily formed definitions (of straight lines, parallels, etc.)—which, observe, are creations of the intellect worked up from sense-data, mere conceptions of the understanding—he cannot deductively prove from the definition of the abstract geometrical figure, termed a rectangle, that its area can be got by multiplying together the numbers measuring the lengths of its sides, provided they have a common measure, while, if they have not a common measure, a product can be obtained giving the result to any degree of precision required. Observe that incommensurability is not a property of objectively existent lines; it can logically be proved of, and therefore applied to, only ideal geometrical creations. Hence the glory of the Pythagorean school of mathematics—the creation of the theory of incommensurable magnitudes.

So far all is pure theory; the corresponding geometrical figures exist only in the imagination, as ideas of the man's mind; they are simply conceptions. In applying these to concrete, visible surfaces, our geometrician foresees that the so-called sides of the objectively existent rectangle he wishes to measure cannot possibly be more than rough approximations to his ideally defined straight lines (*e.g.*, they must have breadth, or he could not see them); that the surface of the rectangle, that the angles, etc., are but rough copies of his geometrical plane surface, right angles, etc. But, although this is so, such facts simply serve to exhibit the excellence of his ideal geometry for purposes of application to the concrete; since, however closely approaching straightness lines may be actually drawn, and however nearly plane surfaces may be actually made on matter, the geometrical theorems, being based on lines defined by man's own creative thought as perfectly straight, and on plane surfaces that are similarly defined as perfectly plane, etc., are thereby efficient to cope with any kind of physical measurement, however precise it may become. Indeed, the absolute precision of geometrical science ever offers an ideal towards which actual physical measurement may strive, but which it can, obviously, never reach, though ever approaching nearer. In this aspect geometry has analogy with moral law,