

GEOMETRICAL SOLUTIONS

OF THE

QUADRATURE OF THE CIRCLE.

LEMMA 1.

With any radius CA, describe the semicircle ABDE, and make the distances AB and BD each equal to the radius AC, also describe from A through C the arc BC, and from B the arc ACD, and bisect the arc CD in the point D', and join A and D. Then from any number of points a, b, c, d, e, f, g, h, of the arc BD make on the arc BC, the arc Ba' equal to the arc Ba, the arc Bb' equal to Bb, Bc' equal to Bc, &c., and from the point A as a center with the radius Aa, describe the arc aa', and from C with the distance Ca' describe the arc a'm, intersecting the arc aa' in the point m. In the same manner from the centers A and C, through the points b and b', c and c', &c., describe the intersections n, o, p, q, r, s, t, &c., and through the points B, m, n, o, &c., draw the line Bmnopqrst, &c., which let be granted is drawn through infinity of points of intersection, and cutting the arc CD—It shall be a curved line passing through the point D'.

For make the arc BI equal to the arc BB'; but the arc CD' is by construction equal to the arc CB', and BB' is equal to CD'—hence the point D', must be on the intersection of the arcs B'D' and ID'—consequently the point D' must be on the line Bmnopqrst, &c., but the radii of each intersection are unequal, and, each intersection is of different radii—therefore the line Bmnopqrst, &c., must be a curved line, cutting CD in D'.

LEMMA 2.

From the point A as a center through B, describe the arc BCA'' and from D' as a center describe through B, the arc HBA'A'' meeting the arc BCA'' in the point A''—and from A'' as a center describe the arc AD', cutting the arc BA' in the point C', and the arc BC in the point B', and join A', passing through the intersections B and H by construction. Next make the arcs Ba and Ba' equal to each other, and Bb' equal to Bb, Bc' equal to Bc, and Bd' equal to Bd, &c. Also make D'a' equal to D'a, D'b' equal to D'b, D'e' equal to D'e, and D'd' equal to D'd, &c., and through the points a and a', b and b', c and c', and d and d', &c., describe from C and A as centers, the intersections m, n, o, p, &c. In the same manner from C' and A'' as centers describe the intersections m'n'o'p'—it is evident, that a line drawn through the points D', m', n', o', p', &c., and B, m, n, o, p, &c., must meet in a common point G, on the line A'B'H—for the arcs B'H and B'B' are symmetrical with the arcs D'H and D'B' to the straight line A'B'H. The points B, m, n, o, p, ... G, and the points D', m', n', o', p', ... G shall be on the arc of a circle.

For let G be the point of meeting of the curve lines Bmnop and D'm'n'o'p', &c., on the straight line A'B'H and take any point K* on the line GH, at less distance from C than the half of GH, or near to G—and through the points B, K, and D', describe the arc BKD',—also make the distance GL, equal to GK, and through the points B, L, and D', describe the arc BLD'.—Next from the center A, with the distance AG, describe the arc Gr, meeting the arc BH in the point r and cutting the arc BK in the point g',—also from C, describe the arc Gn, and from A through any point of the arc Br, describe the arc pn, cutting the arc BK in the point n'—Again make Bm equal to Bp, and from C as a center, describe the arc nm, cutting the arc BL in the point n'', and intersecting the arc pn in the point n—also from C as a center through the points g' and n', describe the arcs g'x and n's, and from A as a center through the points g and n'', describe the arcs gq and n''o.

Now let KG and LG, each be bisected, and through the points of each bisection and the points B and D', describe circular arcs—the one arc must be between the arcs BGD' and BKD' and the other arc between the arcs BGD' and BLD', hence it is