

in words, instead of in figures, either after or below the numerators, the theory of fractions would be mastered with very little more trouble than that of whole numbers. But when the pupil meets such forms as  $\frac{2}{3}$ ,  $\frac{10}{13}$ , &c., his ideas of numbers lead him to confound the terms in spite of abstract rules and definitions, and unless thoroughly enlightened, all fractional operations will long be regarded by him as mysterious and unreasonable. It is not sufficient for him to learn that "A fraction denotes one or more of the equal parts of a unit," that "the lower number is called the Denominator, and shows into how many equal parts the unit is divided," that "the upper is called the Numerator, and shows how many of such equal parts are taken to form the fraction," or that "A fraction also represents the quotient of the numerator by the denominator." These abstractions are good enough when the pupil is prepared for them; but in order to present the subject in an intelligent and simple form, his knowledge of integral numbers must be appealed to, and the relation existing between the integer and fraction must be investigated. Unity or the integer is the basis of all arithmetical calculation. Whole numbers reckon up from fractional numbers, up to this standard. In whole numbers we can tell the numerical value of any combination of figures, because we have a fixed scale of notation, and we know that any figure is increased tenfold every place it is removed to the left of a certain point, known as the decimal point, and marking the place of units. In other words, we know that ten units of any order make one of the next higher. Fractional numbers that have a decimal scale of notation, that is, fractions whose figures decrease in value tenfold, as they recede from the decimal point, may be written like whole

numbers. Thus the mixed number,  $24\frac{3}{10}$ , may be written 24.3 and we know from the position of these figures, that the 2 represents twenty units or two of the order of tens, the 4, four units and the 3, three units of the order of tenths. As fractions, strictly speaking, reckon up to unity, they must have a unit or standard to reckon from. The fractional unit is not fixed in value like the integral; but on the contrary its numerical value is as various as the equal parts into which unity may be divided, hence the complicated forms in which Vulgar Fractions are written. If it were customary to write integral numbers in different scales of notation, we would have to indicate the scale in which any number was written in order to know how many units of the lower order would make one of the next higher. This is exactly the case in fractions. Unity is the next higher order to all of them and, as they have different scales of notation, we have to employ the denominators to indicate how many fractional units make one of the order of integers. The fractional unit is, in every instance, one of the equal parts into which the integral unit is divided. Let the

B            C

line A—————D represent unity, and let it be divided into three equal parts AB, BC and CD; each of these is one-third of the unit AD, and we may use any one of them, say AB as the fractional unit, which is therefore  $\frac{1}{3}$ . Any two of these parts, AC represents  $\frac{2}{3}$  of AD, that is, two fractional units, three of which make the integral unit AD. Strictly speaking a fraction does not denote the quotient of the numerator by the denominator; but the number of times the quotient of unity by the denominator is reckoned, although by deduction, we know that  $\frac{2}{3} = 2 \div 3$ .