SCHOOL WORK.

MATHEMATICS.

ARCHIBALD MACMURCHY, M.A., TORONTO, EDITOR.

EDUCATION DEPARTMENT, TORONTO.

JULY EXAMINATIONS, 1886.

First Class Candidates—Grade C.

TRIGONOMETRY.

Examiner—J. A. McLellan, LL.D.

1. Define *circular measure.* Assuming $\frac{22}{7}$ to be the circular measure of two right angles, express in degrees the angle whose circular measure is θ.

Express the angle $\frac{1}{1000}$ in degrees, etc.
Ans. 3′. 26″. 3.

2. Define the cosine of an angle, and trace the changes in the value and sign of the cosine as the angle increases from 0° to 270°.

Find tan A from the equation $\sin^2 A + 5\cos^2 A = 3$. *Ans.* $\tan A = \pm 1$.

3. Prove geometrically that $\sin(A-B) = \sin A \cos B - \cos A \sin B$, A and B being positive angles less than 90°.

4. Prove the following relations—

(1) $\tan^2 A - \sin^2 A = \tan^2 A \sin^2 A$.

(2) $\tan A = \dfrac{\sin A + \sin 2A}{1 + \cos A + \cos 2A}$

(3) $\dfrac{\cos(x-3y) - \cos(3x-y)}{\sin 2x + \sin 2y} = 2\sin(x-y)$

4. (3) $\dfrac{\cos(x-3y) - \cos(3x-y)}{\sin 2x + \sin 2y} =$

$$\frac{\cos\{2(x-y)-(x+y)\} - \cos\{2(x-y)+(x+y)\}}{2\sin(x+y)\cos(x-y)}$$

$$= \frac{\sin 2(x-y)}{\cos(x-y)} = 2\sin(x-y).$$

5. A tree subtends an angle whose tangent is 2 at a point on the horizontal plane on which it stands, and 90 feet farther off it subtends an angle whose tangent is $\frac{1}{2}$; find the height of the tree. *Ans.* 60 ft.

6. Prove the following—

(1) $\cos a + \cos\beta - \sin a \sin(a+\beta)$
$= 2\cos^2\frac{1}{2}(a+\beta)\cos a$.

(2) $\cos A - \sin A \tan\frac{1}{2}A$
$= \cos 2A + \sin 2A \tan\frac{1}{2}A$.

(3) If $\cot x = n\cot(a-x)$, then
$\sin(a-2x) = \dfrac{n-1}{n+1}\sin a$.

6. Prove (1) $\cos a + \cos\beta - \sin a \sin(a+\beta)$
$= 2\cos^2\frac{1}{2}(a+\beta)\cos a$

$= \cos a + \cos\beta - \sin^2 a\cos\beta - \sin a\cos a\sin\beta$

$= \cos a\{1 + \cos a\cos\beta - \sin a\sin\beta\}$

$= \cos a\{1 + \cos(a+\beta)\} = 2\cos^2\frac{1}{2}(a+\beta)\cos a$

(2) $\cos A - \sin A\tan\frac{1}{2}A$
$= \cos 2A + \sin 2A\tan\frac{1}{2}A$

$= \cos A \cdot \cos\frac{1}{2}A - \sin A\sin\frac{1}{2}A$
$- \cos 2A\cos\frac{1}{2}A + \sin 2A\sin\frac{1}{2}A$

$= \cos(A + \frac{1}{2}A) \quad = \cos(2A - \frac{1}{2}A)$.

(3) If $\cot x = n\cot(a-n)$

then $\sin(a-2x) : \dfrac{n-1}{n+1}\sin a$.

$$\frac{\cos x}{\sin n} = n\,\frac{\cos(a-x)}{\sin(a-x)}$$

$\cos x\sin(a-x) = n\cos(a-x)\sin x$

$\sin\{(a-x)+x\} + \sin\{\overline{a-x}-x\}$

$= n\left[\sin\{\overline{a-x}+x\} - \sin\{\overline{a-x}-x\}\right]$

$\sin a + \sin(a-2x) = n\sin(a-x)$

$\sin(a-2x)$

$\therefore \sin(a-2x) = \dfrac{n-1}{n+1}\sin a$

7. The angle of elevation of a tower of height h is a, how much farther off is the point from which the elevation is $90° - a$.

7. If b is base of △ when a is ∠ of elevation x = dist. off when ∠ of elevation is $\overline{90-a}$, then $\cot a = \dfrac{b}{h}$, $\cot(90-a) = \dfrac{x+b}{h}$

$$\therefore x = h\{\cot\overline{90-a} - \cot a\}$$

$$= h\left\{\frac{h}{b} - \frac{b}{h}\right\} = \frac{h^2 - b^2}{b}.$$

8. From two points A and B in the same vertical plane with a tower, and 66 feet