one is as general as the other. Hence we can assign the meaning of the negative index, for f^{-1} means the reverse operation to f^{1}; if both f^{1} and f^{-1} be performed on x, one undoes what the other does, and the result is x. So that $f^{-1}(x)$ represents that quantity or quantities, for there may be more than one, on which if you perform the function denoted by f^{1} the result is x. And so f^{-2} denotes that quantity or quantities on which if you perform the function f^{1} *twice* the result is x. It will not be possible in every case to assign a numerical or even symbolical expression of every inverse function that may occur, but it appears to me that the *meaning* of the notation is perfectly definite, and that it ought to be treated as such. The theory of indices stands on very different grounds from any arbitrary convenient explanation of, for instance, the symbol $\sqrt{-1}$, derived from the truth of results obtained by treating it as a real quantity. It may, however, be as well in conclusion to notice one or two obvious cases to which the above remarks are applicable:

(1). Theory of Indices in multiplication or division of like quantities in arithmetical algebra,—

Here $a^m = a \times a \times a$ to m factors.

Now a denotes an operation performed on unity, namely, multiplying it by a. Hence a replaces f^{1} and 1 replaces x, 1 being usually for simplicity omitted. Thus $a^0 = a^0\,(1) = 1$.

a^{-1} = a quantity which, multiplied by a, will $= 1$, *i.e.*, $= \frac{1}{a}$.

a^{-2} = a quantity which, multiplied twice by a, will give 1, *i.e.*, $= \frac{1}{a^2}$, and so on.

Unity is here abstract or concrete, and the result abstract or concrete accordingly. In the few cases in which an interpretation may with more or less strictness be applied to the multiplication or division by one another of concrete magnitudes, the unit will of course be of that denomination which is denoted by the index after such multiplication or division.

(2). Indices denoting Trigonometrical Functions, for example,—

Sin$^{0}(x)$ means x.
Sin (x) " the sine of x.
Sin$^{-1}(x)$ " that angle of which the sine is x.
Sin$^{2}(x)$ " the sine of the sine of x, and so on.