$\therefore$ value required is $\dfrac{mcx^2}{a^2(m+1)} + \dfrac{cx^{\frac{3}{2}}}{b^{\frac{3}{2}}(m+1)}$

6. Insert n arithmetical means between two given terms a and b.

There are n arithmetical terms between 1 and 31, such that the 7th mean : $(n-1)^{th}$ mean $= 5:9$; find n.

If a, b, c be the p^{th}, q^{th}, r^{th} terms respectively of an arithmetic series; show that

$$a(q-r)+b(r-p)+c(p-q)=0.$$

6. (1) Text book.

(2) Let d = common difference.

$$1+(n+1)d=31\,;\ \therefore d=\frac{30}{n+1}\,;$$

also $1+7d : 1+(n-1)d :: 5:9$.

From these equations $n=14$.

Let $m=$ 1st term, and we have the equations $m+(p-1)d=a$, $m+(q-1)d=b$,

$$m+(r-1)d=c$$

$$(2)-(3) \text{ gives } d(q-r)=b-c,$$
$$(3)-(1) \quad " \quad d(r-p)=c-a,$$
$$(1)-(2) \quad " \quad d(p-q)=a-b.$$

Multiplying these three equations by a, b and c respectively, and adding

$$d\{a(q-r)+b(r-p)+c(p-q)\}=0.$$

as d is not zero, $a(q-r)+b(r-p)+c(p-q)=0$.

7. Find the sum of a given number of quantities in Geometrical Progression, the first term, and the common ratio being supposed known. Find also the sum of the same series to infinity.

If P be the continued product of n quantities in Geometrical Progression, S their sum, and S_1 the sum of their reciprocals; show that $P^2=\left(\dfrac{S}{S_1}\right)^n$

7. (1) Text book.

(2) $P=a \cdot ar \cdot ar \cdot \ldots \ldots ar^{n-1}=a^n r^{\frac{1}{2}n(n-1)}$

$P^2=a^{2n}r^{n(n-1)}$

$$S=\frac{a(r^n-1)}{r-1},$$

$$S_1=\frac{\frac{1}{a}\left\{\left(\frac{1}{r}\right)^n-1\right\}}{\frac{1}{r}-1}=\frac{1-r^n}{ar^{n-1}(1-r)}$$

$$\left(\frac{S}{S_1}\right)^n=(a^2r^{n-1})^n=a^{2n}r^{n(n-1)}=P^2.$$

8. Given M and N the m^{th}, and n^{th} terms of a Harmonical Progression; find the $(m+n)^{th}$ term.

The sum of three numbers in Harmonical Progression is 26, and the product of the extremes exceeds the square of the mean by the mean, find the numbers.

8. (1) Let a be the first term of the series inverted, d the common difference

$$a+(m-1)d=\frac{1}{M},\ a+(n-1)d=\frac{1}{N},$$

$$d=\frac{N-M}{MN(m-n)}.$$

The $(m+n)^{th}$ term of this series is

$$a+(m+n-1)d=\frac{1}{M}+nd=\frac{mN-nM}{MN(m-n)}\,;$$

$\therefore$ the $(m+n)^{th}$ term of the H. series

$$=\frac{MN(m-n)}{mN-nM}.$$

(2) Let x, y and z be the three nos. in H P.

$$y=\frac{2xz}{x+z},\ x+y+z=26,\ xz=y^2+y.$$

$$\text{From } (1)+(2)-(3)\ x+z=\frac{2(y^2+y)}{y}=26-y$$

whence $x=6$, $y=8$, $z=12$.

9. Find the number of permutations of n things taken r at time.

Given m things of one kind, and n things of another kind, find the number of permutations that can be formed containing r of the first and s of the second.

9 See Todhunter, Art. 490 and problem 17, in the exercises immediately following.

10. Assuming the Binomial Theorem for positive integral indices, prove it for fractional and negative indices.

Show that $\left(\dfrac{1+2x}{1+x}\right)^n$

$$=1+n\left(\frac{x}{1+2x}\right)+\frac{n(n+1)}{1.2}\cdot\left(\frac{x}{1+2x}\right)^2+\text{etc.}$$

Find the greatest term in the expansion of

$$\left(1+\frac{5}{6}\right)^{\frac{3}{2}}$$

10. $\left(\dfrac{1+2x}{1+x}\right)^n=\left(\dfrac{1+x}{1+2x}\right)^{-n}$

$$=\left\{1-\frac{x}{1+2x}\right\}^{-n}=1+n\left(\frac{x}{1+2x}\right)+\ldots$$

(2) The 2nd.