$$= -s^2 + (ab+ac+bc)s - 4rRs,$$

$$\therefore r^2+s^2+4Rr = ab+ac+bc$$

$$a+b+c=2s,$$

$\therefore$ the values of a, b and c are contained in the cubic

$$x^3 - 2sx + (r^2+s^2+4Rs)x - 4Rrs = 0.$$

Solution of 161 by the proposer, D. F. H. WILKINS, B.A., Mathematical Master, High School, Chatham.

161. If A, B, C be the angles of a plane triangle, prove that

$$\begin{vmatrix} \sin 2A, & 0, & \sin 2B, & \sin 2C \\ 0 & \sin 2A, & \sin 2C, & \sin 2B \\ \sin 2C, & \sin 2B, & 0, & \sin 2A \\ \sin 2B, & \sin 2C, & \sin 2A, & 0 \end{vmatrix}^{\frac{1}{2}}$$

$$= 2\sin 2A \sin 2B \sin 2C.$$

Expanding the determinant, we have

$$\{2\sin^2 2A \sin^2 2B + 2\sin^2 2B \sin^2 2C + 2\sin^2 2C \sin^2 2A - \sin^4 A - \sin^4 B - \sin^4 C\}^{\frac{1}{2}}$$

$$= \{(\sin 2A + \sin 2B + \sin 2C)(\sin 2A + \sin 2B - \sin 2C)(\sin 2B + \sin 2C - \sin 2A)(\sin 2C + \sin 2A - \sin 2B)\}^{\frac{1}{2}}$$

$$= \{(4\sin A \sin B \sin C)(4\sin C \cos A \cos B)(4\sin A \cos B \cos C)(4\sin B \cos C \cos A)\}^{\frac{1}{2}}$$

$$= \{4(4\sin^2 A \cos^2 A)(4\sin^2 B \cos^2 B)(4\sin^2 C \cos^2 C)\}^{\frac{1}{2}}$$

$$= 2\{\sin^2 2A \sin^2 2B \sin^2 2C\}^{\frac{1}{2}}$$

$$= 2\sin 2A \sin 2B \sin 2C.$$

Cambridge Mathematical Tripos, 1880 (found in September Number). Solutions by F. BOULTBEE, U. C.

4. An A. P., a G. P. and an H. P. have a and b for their first two terms: shew that the $(n+2)^{th}$ terms will be in G. P. if

$$\frac{b^{2n+2} - a^{2n+2}}{ba(b^{2n} - a^{2n})} = \frac{n+1}{n}.$$

$(n+2)^{th}$ term of A. P. $= a + (n+1)(b-a)$,

$(n+2)^{th}$ term of G. P. $= \left(\frac{b}{a}\right)^{n+1} \cdot a$,

$(n+2)^{th}$ term of H. P. $= \dfrac{ab}{b+(n+1)(a-b)}$;

these are in G. P. if

$$\frac{a+(n+1)(b-a)}{1} \times \frac{ab}{b+(n+1)(a-b)}$$

$$2\left\{\left(\frac{b}{a}\right)^{n+1} \cdot a\right\}^2;$$

that is if $\dfrac{b^{2n+2} - a^{2n+2}}{ba(b^{2n} - a^{2n})} = \dfrac{n+1}{n}$.

6. There are n points in a plane, no three of which lie in a straight line. Find how many closed r-sided figures can be formed by joining the points by straight lines.

Number of figures equal combinations of n things, taken r at a time $= \dfrac{\lfloor n}{\lfloor r \lfloor n-r}$, since no three of the points are in a straight line.

(vii.) If an arc of ten feet on a circle of eight feet diameter subtend at the centre an angle $143^\circ\ 14'\ 22''$, find the value of π to four decimal places.

$$\frac{10}{8\pi} = \frac{143^\circ\ 14'\ 22''}{360^\circ} = \frac{515662}{1620000},$$

$$\therefore \pi = 3{\cdot}1415.$$

(x.) Establish the identity

$$\tan\frac{x+y}{2}\tan\frac{x-y}{2}$$

$$= \frac{\operatorname{cosec} 2x \operatorname{cosec} y - \operatorname{cosec} 2y \operatorname{cosec} x}{\operatorname{cosec} 2x \operatorname{cosec} y + \operatorname{cosec} 2y \operatorname{cosec} x}.$$

$$\frac{\dfrac{1}{\sin 2x \sin y} - \dfrac{1}{\sin 2y \sin x}}{\dfrac{1}{\sin 2x \sin y} + \dfrac{1}{\sin 2y \sin x}}$$

$$= \frac{\sin 2y \sin x - \sin 2x \sin y}{\sin 2y \sin x + \sin 2x \sin y}$$

$$= \frac{2\sin y \cos y \sin x - 2\sin x \cos x \sin y}{2\sin y \cos y \sin x + 2\sin x \cos x \sin y}$$

$$= \frac{\cos y - \cos x}{\cos y + \cos x} = \frac{2\sin\dfrac{x+y}{2}\sin\dfrac{x-y}{2}}{2\cos\dfrac{x+y}{2}\cos\dfrac{x-y}{2}}$$

$$= \tan\frac{x+y}{2}\tan\frac{x-y}{2}.$$

PROBLEMS.

165. Prove that the equations

(1) $x+y+z = a+b+c$,

(2) $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$,