MATHEMATICS.

Horner's Synthetic Division.

(From the Quarterly for July, 1878.)

The following explanation of Horner's Division exhibits an easy mode of transition from the ordinary to the synthetic method.

Let it be required to divide
$x^4 + x^3 - 5x^2 + 15x - 8$ by $x^2 - 2x + 3$

Performing this division in the usual way we get .

$$\begin{array}{rl}
 & (x^2 + 3x - 2 \\
x^2 - 2x + 3) & x^4 + x^3 - 5x^2 + 15x - 8 \\
 & \underline{x^4 - 2x^3 + 3x^2} \\
 & 3x^3 - 8x^2 \\
 & \underline{3x^3 - 6x^2 + 9x} \\
 & -2x^2 + 6x \\
 & \underline{-2x^2 + 4x - 6} \\
 & 2x - 2
\end{array}$$

The first term in each subtrahend, being always the same as the first term in each minuend, may therefore be omitted, thus:

$$\begin{array}{rl}
 & (x^2 + 3x - 2 \\
x^2 - 2x + 3) & x^4 + x^3 - 5x^2 + 15x - 8 \\
 & \underline{-2x^3 + 3x^2} \\
 & 3x^3 - 8x^2 \\
 & \underline{-6x^2 + 9x} \\
 & -2x^2 + 6x \\
 & \underline{4x - 6} \\
 & 2x - 2
\end{array}$$

Now, if we bear in mind that subtraction in Algebra is effected by changing the signs of the terms in the subtrahend, and then adding together the subtrahend and minuend, it will readily be seen that if the signs of those terms in the divisor, which produce the subtrahend, be changed, the subtrahend will then be formed ready for adding, and no change of sign will be required. The sign of the first term in the divisor is not to be changed, as it helps to form no part of any of the subtrahends now used. The process of division will then be as follows:—

$$\begin{array}{rl}
 & (x^2 + 3x - 2 \\
x^2 + 2x - 3) & x^4 + x^3 - 5x^2 + 15x - 8 \\
 & \underline{2x^3 - 3x^2} \\
 & 3x^3 - 8x^2 \\
 & \underline{6x^2 - 9x} \\
 & -2x^2 + 6x \\
 & \underline{-4x + 6} \\
 & 2x - 2
\end{array}$$

Here two terms ($-8x^2$, $6x$) occur, which may be advantageously omitted. The first of these is the sum of $-5x^2$ and $-3x^2$; to this sum is added $6x^2$ and thus the term $-2x^2$ is obtained as the sum of $-5x^2$, $-3x^2$ and $6x^2$ The term $-8x^2$ is therefore quite unnecessary, and so also is the term $6x$ Omitting these quantities, the operation of dividing would then be this:—

$$\begin{array}{rl}
 & (x^2 + 3x - 2 \\
x^2 + 2x - 3) & x^4 + x^3 - 5x^2 + 15x - 8 \\
 & 2x^3 - 3x^2 \\
 & \underline{3x^3} \\
 & 6x^2 - 9x \\
 & \underline{-2x^2} \\
 & \underline{-4x + 6} \\
 & 2x - 2
\end{array}$$

This may be more compactly arranged, thus:—

$$\begin{array}{rllll}
 & & & & (x^2 + 3x - 2 \\
x^2 + 2x - 3) & x^4 + x^3 & -5x^2 & + 15x & - 8 \\
 & & 2x^3 & - 3x^2 & - 9x \\
 & & 3x^3 & 6x^2 & \underline{-4x + 6} \\
 & & & -2x^2 & 2x - 2
\end{array}$$

New, since in adding any number of terms together the order in which they are taken is immaterial, the third and fourth columns may be rearranged thus:—