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Formulae for One-Story and Two-Story Bents

Elastic Deformation Method Used for Exact Determination of Stresses in Simple Types of Structures—Equations Derived Analytically to Save Time for Designing Engineers Engaged in Steel or Reinforced Concrete Construction

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WHEN designing one-story and two-story bents, without diagonals, subject to horizontal forces, engineers usually base their design upon some assumption as to the location of the points of contraflexure of the columns.

Following such assumption, the bending moments produced in the columns by the horizontal forces are easily calculated. It is evident, however, that the location of the points of contraflexure is dependent upon the relative moments of inertia of the columns and struts, since the stiffness of the struts as compared to the columns determines the amount of deflection at the joints. The results obtained by

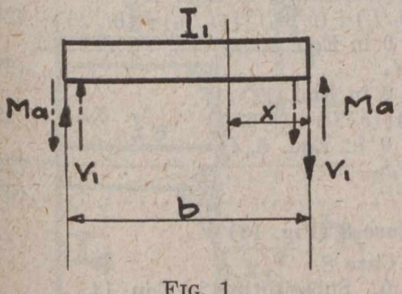

FIG. 1

this method of calculation are, of course, only approximate.

In this article, an analysis, based upon the elastic theory of structures, is made in order to determine the bending moments and the location of the points of contraflexure in the columns of a two-story bent. From the formulæ thus derived, the ten different cases illustrated by Figs. 6 to 15 are deduced.

If the structure be subject to vertical forces as well as horizontal, it is but necessary to make an algebraical addition of the stresses produced by the different forces.

In this article the following assumptions are made:—

(1)—That each column takes one-half the load.

(2)—That the connection of the column to the strut is rigid.

(3)—That the foundations are non-yielding.

All dimensions are expressed in the same units, and the following nomenclature is used:—

P_1 and P_2 Horizontal forces.

h_1 and h_2 and b Distances as shown in figures.

I_1 and I_3 Moment of inertia of upper and lower girder, respectively.

I_2 and I_4 Moment of inertia of upper and lower column, respectively.

M_a Moment at junction of upper column and upper girder.

M_b Moment at junction of upper column and lower girder.

M_c Moment at junction of lower column and lower girder.

M_d Moment at base of column.

x_1 and x_2 Distances to points of contraflexure.

Referring to Fig. 2, the moments about E may be expressed as follows:—

$$P_1h_1 = 2M_b + V_1b.$$

Solving for V_1,

$$V_1 = (P_1h_1 - 2M_b)/b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Again referring to Fig. 2, the moments about D may be expressed as follows:—

$$P_1(h_1+h_2) + P_2h_2 = 2M_d + V_2b.$$

Solving for V_2,

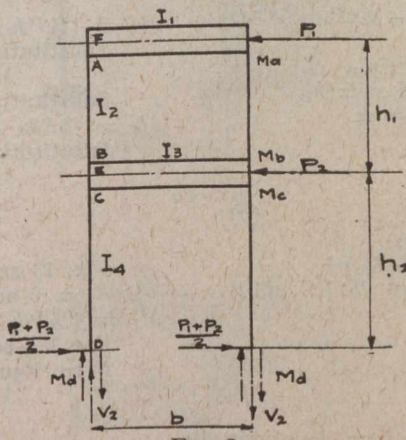

FIG. 2

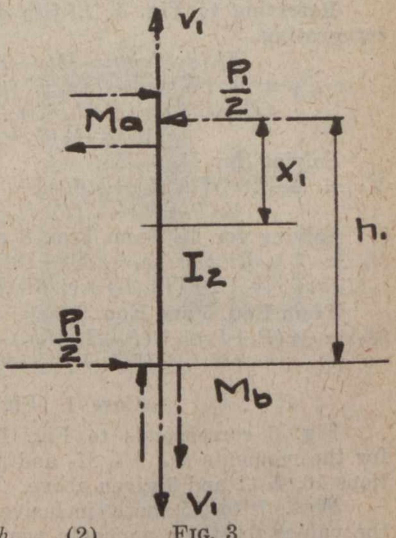

FIG. 3

$$V_2 = [P_1(h_1+h_2) + P_2h_2 - 2M_d]/b \ldots\ldots (2)$$

Then,

$$V_2 - V_1 = [(P_1+P_2)h_2 - 2M_d + 2M_b]/b \quad \ldots\ldots (3)$$

Referring to Fig. 1, the moments about F are

$$V_1b = 2M_a.$$

Solving for M_a and substituting the value for V_1 from Equ. 1,

$$M_a = (P_1h_1/2) - M_b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Referring again to the moments about E (see Fig. 2),

$$(P_1+P_2)h_2 - 2M_d = 2M_c.$$

From this,

$$M_c = \tfrac{1}{2}(P_1+P_2)h_2 - M_d \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The equation of the elastic curve (see Fig. 1) is

$$EI_1(d^2y/dx^2) = V_1x - M_a.$$

By integrating,

$$EI_1(dy/dx) = \tfrac{1}{2}V_1x^2 - M_ax + C_1.$$

When $dy/dx = (dy/dx)_A$, and $x = 0$,

$$C_1 = EI_1(dy/dx)_A.$$

By substituting in above,

$$EI_1(dy/dx) = \tfrac{1}{2}V_1x^2 - M_ax + EI_1(dy/dx)_A.$$

Integrating again,

$$EI_1y = \tfrac{1}{6}V_1x^3 - \tfrac{1}{2}M_ax^2 + EI_1(dy/dx)_Ax + C_2.$$