And therefore the velocity s_x with which the water level reaches the ideal overflow elevation is:

$$s_x = -.00805\, e^{-\frac{520}{165}} + .0810\, e^{-\frac{520}{15.65}} = -.000346 \frac{\text{feet}}{\text{sec.}}$$

With these initial values the movement in the third period may be determined. Because we assumed that the spillway crest is at the elevation of the static level $n - n$, the overflow period goes further but with smaller fluctuations of water level and overflow quantities.

If the spillway is built not in the surge tank, but in the main conduit, the principal equation requires a supplement. According to Fig. 10, a shaft at the distance L' from the beginning of the conduit is driven down to the conduit with a section A', through which water from the conduit goes over a spillway whose ideal overflow crest is as before the distance E from the static level $n - n$. At the distance L'' is the surge tank with the section A''. In the hydraulic equilibrium with Q_1 cubic feet per second discharge through the penstocks, the water surface in the overflow shaft will be below the static level $n - n$ by the distance $h_1' = n'.v_1$; in the surge tank by $h_1'' (n' + n'') v_1$. v_1 is the velocity in the conduit of the area a which corresponds to the discharge Q_1. n' and n'' are the function coefficients corresponding to the distances L' and L''. For flow fluctuations the distance z_1 and z_2 from the static level $n - n$ at the velocities v_1' and v_1'' and the water surfaces in shaft and surge tank will have different levels and there will be overflow in the shaft as soon as the water surface reaches the spillway crest. It is sufficient to carry through the solution of the principal equation for the latter period and to handle the first period as a special case of it.

For the purpose of simplifying the work, we consider only the case of sudden total shut-down. For both parts of the conduit, we get the following movement equations:

$$\frac{L'\, dv_1'}{g\, dt} + z_1 + n'.v_1' = o$$

$$\frac{L''\, .dv_1''}{g.dt} + (z_2 - z_1) + n''\, v_1'' = o \qquad (86)$$

The equations which express continuity follow from the consideration that from the upper conduit in a unit of time dt, a quantity of water must flow into the shaft which is equal to the algebraic sum of—

1st—the quantity which flows away in the lower conduit, which is equal to $a.v_1''.dt$.

2nd—the simultaneous filling of the shaft with the quantity $A'.v_1'.dt$.

3rd—the simultaneous overflow quantity $k\,(z_1 - E)\, dt$. But the water quantity which flows through the lower conduit in the time dt is also equal to the simultaneous filling $A''\, v_1''$ dt in the surge tank. The following two equations express, therefore, the continuity

$$a.v_1' = a.v_1'' + A'\, s_1 + k\,(z_1 - E)$$

and

$$av_1'' = A'' s_2 \qquad (87)$$

The motion equations and the second equation for continuity reach their values before as well as after the overflow on the spillway. The second equation for continuity is correct for $k = o$, for periods without any overflow. If we consider that $s_1 = \frac{dz_1}{dt}$ and $s_2 = \frac{dz_2}{dt}$ we may, with the aid of the continuity equations, eliminate the velocities v_1', v_1'' and their derivations. We get then two simultaneous differential equations of the second order, from which we eliminate again z_1 and its derivations, whereas for the determination of z_2 we get a linear differential equation of the fourth degree with constant coefficients, the integration of which does not involve any great difficulty.

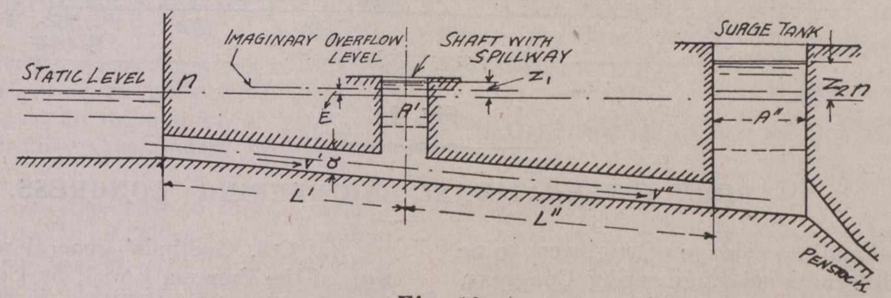

Fig. 10.

In order to make the spillway especially efficient, we manage it so that the spillway crest lies below the static level $n - n$, and this in a distance which is equal to or a little less than $h_1' = n'.v_1$. In this case, after the shutdown is finished, a very rapid overflow will occur in the spillway and will continue, and finally there exists a constant flow over the spillway, whereby naturally as much flows through the upper conduit as goes away over the spillway and where the water level in the shaft, as well as in the surge tank lies at a distance under $n - n$, which corresponds to the hydraulic slope necessary for the flow through the upper conduit.

If in this case A' and the spillway widths are small enough with reference to the hydraulic slope, to keep the preceding fluctuations so near the static level of the water surface in the shaft, that the variation of the inflow to the shaft may be neglected, the problem becomes simplified, because the first motion equation drops out and in the first equation for continuity, the values $a.v_1'$ become constant and equal to q.

Considering that $s_1 = \frac{dz_1}{dt}$ and $s_2 = \frac{dz_2}{dt}$, we get from the first equation of continuity (87)