The first order necessary condition is obtained by maximizing (4) with respect to R as follows:

$$\frac{1}{\rho}B'\psi = s. \tag{5}$$

The condition (5) states that the inventor will spend resources up to that level where present value of the income derived from an additional unit of research is equal to its unit cost. To determine the optimal or welfare maximizing patent term one maximizes (3) subject to (5). Differentiating (3) with respect to ψ, noting from equation (5) that

$$\partial R/\partial\psi = B'(R)/B''(R)\psi > 0,$$

we obtain

$$\frac{\partial W}{\partial\psi} = -\frac{B'^2}{\rho B''\psi} - \frac{\eta}{2\rho}\left[\frac{2BB'^2}{B''\psi}(1-\psi) + B^2\right] + s\frac{B'}{B''\psi} = 0. \tag{6}$$

Substituting $\rho s = B'\psi$, equation (5) in equation (2.6), and solving for ψ, one obtains:

$$\psi^*_M = \frac{1+\eta B}{1+\eta B(1+\frac{k}{2})}, \tag{7}$$

where $k = -B''B/B'^2 > 0$ is the degree of concavity of B(R).

Nordhaus calls (5) the inventor's equilibrium and (7) the policy maker's equilibrium. The optimal patent term is the intersection of these two curves. The optimal patent life T is given by

$$T = -\frac{1}{\rho}\ln(1-\psi^*),$$

where T ranges from 0 to ∞ as ψ ranges from 0 to 1 and ψ^* is the optimal value of ψ satisfying equations (5) and (7).

The optimal value of T is determined by the intersection of the inventor's equilibrium (5) and the policy maker's equilibrium (7). Treating ρ and η as parameters, we have two equations to solve for B and T simultaneously. Figure 2 shows the equilibrium for two hypothetical curves.