10. Sum the series

$$\frac{1^2}{\lfloor 2}+\frac{2^2}{\lfloor 4}+\frac{3^2}{\lfloor 6}+\ldots\text{to infinity.}$$

Let $4S=4\left\{\frac{1^2}{\lfloor 2}+\frac{2^2}{\lfloor 4}+\ldots+\frac{n^2}{\lfloor 2n}+\ldots\right\}$,

$$\frac{4n^2}{\lfloor 2n}=\frac{2n(2n-1)}{\lfloor 2n}+\frac{2n}{\lfloor 2n}=\frac{1}{\lfloor 2n-2}+\frac{1}{\lfloor 2n-1},$$

$$\therefore 4S=\frac{1}{\lfloor 0}+\frac{1}{\lfloor 1}+\frac{1}{\lfloor 2}+\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+\ldots=e. \quad \therefore S=\frac{e}{4}.$$

Sum the series

$$\tan^{-1}\frac{4}{1.5}+\tan^{-1}\frac{6}{5.11}+\tan^{-1}\frac{8}{11.19}=\ldots\text{ to } n \text{ terms.}$$

This question appears to be erroneous. We suggest that it should be

Sum the series

$$\tan^{-1}\frac{4}{1+1.5}+\tan^{-1}\frac{6}{1+5.11}+\tan^{-1}\frac{8}{1+11.19}+\ldots\text{to } n \text{ terms.}$$

$$\tan^{-1}\frac{4}{1+1.5}=\tan^{-1}\frac{5-1}{1+1.5}=\tan^{-1}5-\tan^{-1}1$$

$$\tan^{-1}\frac{6}{1+5.11}=\tan^{-1}11-\tan^{-1}5,$$

etc. = etc.,

$$\tan^{-1}\left\{\frac{2(n+1)}{1+(n^2+n-1)\left\{(n+1)^2+(n+1)-1\right\}}\right\}$$
$$=\tan^{-1}\left\{(n+1)^2+(n+1)-1\right\}-\tan^{-1}(n^2+n-1),$$

$$\therefore S_n=\tan^{-1}\left\{\frac{n(n+3)}{(n+1)(n+2)}\right\}.$$

11. P, Q, R, S are the middle points of the sides of a quadrilateral taken in order; the intersection of PR and QS lies in the same straight line with the points which bisect the diagonals of the quadrilateral.

Let $ABCD$ be the quadrilateral, take O point of intersection of AB and CD produced as origin, and let

$$\frac{x}{2a}+\frac{y}{2b}=1 \text{ be =n of } AD,$$

$$\frac{x}{1a^1}+\frac{y}{2b^1}=1 \text{ =n of } BC.$$

Then $U.V$ middle points of diagonals are (a, b^1) and $(a^1 b)$. Also, T the intersection of PR and QS is $\frac{a+a^1}{2}, \frac{b+b^1}{2}$; $\therefore T$ lies on the right line UV, and bisects it.

12. An ellipse is inscribed in any triangle, and the polars of the middle points of the sides are drawn; the triangle formed by the three polars is of constant area.

If S be the area of the triangle formed by the three lines, $a_1x+b_1y+c_1=0$,

$$a_2x+b_2y+c_2=0, \quad a_3x+b_3y+c_3=0;$$

then

$$2S=\frac{(a_1b_2c_3)^2}{(a_2b_3)(a_3b_1)(a_1b_2)}. \quad \text{(Salmon's Conics, § 39, 6th ed.)}$$

$(a_1b_2c_3)$ standing for $\begin{vmatrix} a_1, & a_2, & a_3 \\ b_1, & b_2, & b_3 \\ c_1, & c_2, & c_3 \end{vmatrix}$.

and so for (a_2b_3), etc.

Let $\frac{xx_1}{a^2}+\frac{yy_1}{b^2}-1=0$, etc., be the polars of points $P(x_1y_1)$, $Q(x_2y_2)$, $R(x_3y_3)$, with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}-1=0$, and let

$$\Delta=\begin{vmatrix} \frac{x_1}{a^2}, & \frac{y_1}{b^2}, & -1 \\ \frac{x_2}{a^2}, & \frac{y_2}{b^2}, & -1 \\ \frac{x_3}{a^2}, & \frac{y_3}{b^2}, & -1 \end{vmatrix}=-\frac{1}{a^2b^2}\begin{vmatrix} x_1, & y_1, & 1 \\ x_2, & y_2, & 1 \\ x_3, & y_3, & 1 \end{vmatrix}$$

$$=-\frac{2\,[\text{area } PQR]}{a^2b^2}.$$

$$\therefore 2S=\frac{\frac{4}{a^4b^4}(PQR)^2}{\frac{(x_2y_3).(x_3y_1).(x_1y_2)}{a^6b^6}},$$

$$\therefore S=\frac{a^2b^2(PQR)^2}{4(QOR)\,(ROP)\,(POQ)}$$

where O is centre of the ellipse. Now, let