

Theorem 2.1

The equilibria (p^*, q_1^*, q_2^*) and the equilibrium payoffs $E_i^* = E_i(p^*, q_1^*, q_2^*)$, $i = 0, 1, 2$, of the game described above are as follows:

$$(i) \quad \text{If } \frac{d_1}{b_1 + d_1} \cdot \frac{1}{1 - \beta_1} + \frac{d_2}{b_2 + d_2} \cdot \frac{1}{1 - \beta_2} \leq 1, \quad (2.9)$$

$$\frac{d_1}{b_1 + d_1} \cdot \frac{1}{1 - \beta_1} \leq p^* \leq 1 - \frac{d_2}{b_2 + d_2} \cdot \frac{1}{1 - \beta_2}, \quad q_1^* = q_2^* = 0. \quad (2.10)$$

$$E_0^* = E_1^* = E_2^* = 0 \quad (2.11)$$

$$(ii) \quad \text{If } \frac{d_1}{b_1 + d_1} \cdot \frac{1}{1 - \beta_1} + \frac{d_2}{b_2 + d_2} \cdot \frac{1}{1 - \beta_2} > 1, \text{ there are four cases:} \quad (2.12)$$

$$\text{If } \beta_1 < \beta_2 \text{ and } \frac{d_1}{b_1 + d_1} \cdot \frac{1}{1 - \beta_1} \leq 1, \quad (2.13)$$

$$p^* = \frac{d_1}{b_1 + d_1} \cdot \frac{1}{1 - \beta_1}, \quad q_1^* = \frac{1 - \beta_2}{1 - \beta_1}, \quad q_2^* = 1; \quad (2.14)$$

$$\text{If } \beta_1 < \beta_2 \text{ and } \frac{d_1}{b_1 + d_1} \cdot \frac{1}{1 - \beta_1} > 1, \quad (2.15)$$

$$p^* = 1, \quad q_1^* = q_2^* = 1; \quad (2.16)$$

$$\text{If } \beta_1 > \beta_2 \text{ and } \frac{d_2}{b_2 + d_2} \cdot \frac{1}{1 - \beta_2} \leq 1, \quad (2.17)$$

$$p^* = 1 - \frac{d_2}{b_2 + d_2} \cdot \frac{1}{1 - \beta_2}, \quad q_1^* = 1, \quad q_2^* = \frac{1 - \beta_1}{1 - \beta_2}; \quad (2.18)$$

$$\text{If } \beta_1 > \beta_2 \text{ and } \frac{d_2}{b_2 + d_2} \cdot \frac{1}{1 - \beta_2} > 1, \quad (2.19)$$

$$p^* = 0, \quad q_1^* = q_2^* = 1. \quad (2.20)$$

The equilibrium payoffs are obtained by inserting the equilibria into (2.4), (2.5) and (2.6).

Proof

(i) For $q_1^* = q_2^* = 0$ (2.8b) and (2.8c) are equivalent to

$$d_1 - (b_1 + d_1)(1 - \beta_1)p^* \leq 0 \quad \text{and} \quad d_2 - (b_2 + d_2)(1 - \beta_2)(1 - p^*) \leq 0.$$

These lead immediately to the assertion.

(ii) If $\beta_1 < \beta_2$ and $\frac{d_1}{b_1 + d_1} \cdot \frac{1}{1 - \beta_1} \leq 1$, (2.8a) is satisfied iff