MATHEMATICS.

Solutions to Problems in the April Number.

26. Let A be the given point in the circumference; from A draw any two chords AB, AC; through B draw BD parallel to AC and through C draw CE parallel to AB; join DE; through A draw GAH parallel to DE; GAH shall be the tangent required.

Let BD, CE (produced if necessary) meet in K, then BACK is a parallelogram; therefore the angles ABK, ACK are equal; therefore the arcs ABE, ACD are equal, therefore the chords AD, AE are equal. Now suppose F the middle point of DE; join AF; then the angles at F are right angles; therefore AF passes through the centre of the circle. Also, since GAH is parallel to DE, therefore the angles FAG, FAH are right angles; therefore GAH is a tangent to the circle.

27. Let ABC be the given triangle; at the point B make the angle CBD equal two-thirds of a rt. angle; through A draw AD parallel to BC meeting BD in D; join DC; then the triangle DBC is equal to ABC. In BC (produced if necessary) take BE such that the square on BE is equal to the rectangle DB, BC; in BD take BF equal to BE; join FE, then FEB shall be the required equilateral triangle. For since the square on BE is equal to the rectangle DB, BC; therefore DB is to BE as BF is to BC that is the triangles DBC, FBE have one angle of the one equal to one angle of the other and their sides about the equal angles reciprocally proportional; hence they are equal to one another. (Euc. VI. 15.)

28. The series

$$\frac{n}{1\cdot2\cdot3}+\frac{n-1}{2\cdot3\cdot4}+\text{\&c}+\frac{n-(n-1)}{n(n+1)(n+2)}$$

$$=\frac{n}{1\cdot2\cdot3}+\frac{n}{2\cdot3\cdot4}+\text{\&c.}+\frac{n}{n(n+1)(n+2)}$$

$$-\frac{1}{2\cdot3\cdot4}-\frac{2}{3\cdot4\cdot5}-\text{\&c.}-\frac{n-1}{n(n+1)(n+2)}$$

and since the nth term of the first of these series=

$$\frac{1}{n(n+1)(n+2)}=\frac{1}{2n(n+1)}-\frac{1}{2(n+1)(n+2)}$$

$$\therefore\ (n-1)\text{ th term}=\frac{1}{2(n-1)n}-\frac{1}{2n(n+1)}$$

$$(n-2)\text{ th term}=\frac{1}{2(n-2)(n-1)}-\frac{1}{2(n-1)n}$$

$$\text{\&c.}=\text{\&c.}$$

$$\text{3d term}=\frac{1}{2\cdot3\cdot4}-\frac{1}{2\cdot4\cdot5}$$

$$\text{2d term}=\frac{1}{2\cdot2\cdot3}-\frac{1}{2\cdot3\cdot4}$$

$$\text{1st term}=\frac{1}{4}-\frac{1}{2\cdot2\cdot3}$$

Hence by addition we have the sum of the series.

$$=\frac{1}{4}-\frac{1}{2(n+1)(n+2)}$$

$$\therefore\ \frac{n}{1\cdot2\cdot3}+\frac{n}{2\cdot3\cdot4}+\text{\&c.}+\frac{n}{n(n+1)(n+2)}$$

$$=\frac{n}{4}-\frac{n}{2(n+1)(n+2)}$$

Also, the sum of the second series is

$$-\frac{1}{4}+\frac{2n+1}{2(n+1)(n+2)}$$

Therefore the sum of the original series

$$=\frac{n-1}{4}+\frac{n+1}{2(n+1)(n+2)}$$

$$=\frac{n(n+1)}{4(n+2)}$$

29. Since imaginary and surd roots must