When one-half of the arch is loaded, between hinge and crown, the horizontal pressure is

$$H = \frac{pl^2}{16f} \alpha ;$$

and the bending moment at the crown,

$$M = \frac{1}{16} pl^2 (1 - \alpha).$$

The maximum and minimum moments act at the points

$$x = \frac{1}{4} l \frac{3 - 2\alpha}{2 - \alpha} \text{ and } x = \frac{1}{4} l \frac{2\alpha - 1}{\alpha},$$

and have the values

$$\text{max. } M = -\frac{1}{64} pl^2 \frac{(3 - 2\alpha)^2}{2 - \alpha},$$

$$\text{min. } M = -\frac{1}{64} pl^2 \frac{(2\alpha - 1)^2}{\alpha}.$$

Thus, for $\alpha = 1$ the numerical value is $\frac{1}{64} pl^2$.

For the maximum deflection and rise at the crown Müller-Breslau has given the following approximations, the arch being loaded with live load (p per unit length) only:

$$\text{maximum deflection } \frac{pl^4}{EJ} [.00034 + .15 (\frac{i}{l})^2],$$

$$\text{maximum rise } \frac{pl^4}{EJ} [-.00034 + .10 (\frac{i}{l})^2],$$

E being the modulus of elasticity and I the moment of inertia.

The effect of the bending moments and normal forces due to an increase of the temperature of t° can readily be computed as the horizontal pressure produced by it is

$$X_t = \frac{\delta_{at}}{\delta_{aa}},$$

δ_{at} being the movement of the roller end of the auxiliary system for t° increase of temperature, δ_{at} taken positive in the same direction as δ_{aa}.

As $\delta_{at} = EJ \epsilon t l$

$$X_t = EJ \frac{\epsilon t l}{\delta_{aa}} = \frac{15}{8} EJ \alpha \frac{\epsilon t}{f^2}$$

where ϵ is the elongation per unit length for one degree increase.

A yielding of the supports can be treated in a similar way; as already mentioned, it is only a movement in horizontal direction which has influence on the strains and stresses in the arch. A shortening of kl of the distance l between the hinges effects a horizontal pressure

$$X_s = EJ \frac{kl}{\delta_{aa}},$$

or by introducing the value of δ_{aa}

$$X_s = \frac{15}{8} EJ \alpha \frac{k}{f^2}.$$

The Arch Without Hinges.—This type of arch has three statically indeterminate quantities and as those are chosen the normal force X_a, the transverse force X_b, and the bending moment X_c at the crown of the arch, acting from a point O (see Fig. 2) in the axis of symmetry, so situated that each of the three equations, from which the values of the statically indeterminate quantities are calculable, will contain only one of them. Their form will then become similar to that for the horizontal pressure of the arch with two hinges.

$$X_a = \frac{\delta_{ma}}{\delta_{aa}}$$

$$X_b = \frac{\delta_{mb}}{\delta_{cc}}$$

$$X_c = \frac{\delta_{mc}}{\delta_{cc}},$$

the statically determinate auxiliary system being two curved beams with one fixed and one unsupported end.

The "v" forces for the type of arch considered here are:

$$v^a = dx$$

$$v^b = xdx$$

$$v^c = ydx,$$

which again are equivalent to the continuous loads

$$z^a = 1$$

$$z^b = x$$

$$z^c = y.$$

The distance η from the point O to the line AB (Fig. 2) is determined by the equation

$$\eta \int_0^1 Z^a \, dx = \int_0^1 Z^a \, y' \, dx$$

$$\therefore \ \eta = \frac{2}{3} f^\circ,$$

y' being the ordinate of the centre line of the arch measured from AB. The equation for this centre line in the system of co-ordinates, as shown in Fig. 2 with O as origin is then

$$y = \frac{4f}{l^2} (\frac{1}{4} l^2 - x^2) - \eta = f [\frac{1}{3} - 4 (\frac{x}{l})^2].$$

The deflections δ_{ma}, δ_{mb} and δ_{mc} are calculated as moments produced by the loads z, acting on a beam of length l, fixed at the centre and having unsupported ends (Fig. 3). For the left and right half of the arch respectively

$$\delta_{ma} = -\int_{-\frac{1}{2}l}^{x} (x - x_1) \, Z^a dx_1 \text{ and } = -\int_{x}^{-\frac{1}{2}l} (x_1 - x) \, Z^a \, dx_1.$$

In these expressions x is the abscissa of the point for which δ_{ma} is calculated and x_1 the varying abscissa of the