

the income derived from each the same. How much has he finally in three per cent. Stock?

2. Divide $(4x^3 - 3a^2x)^2 + (4y^3 - 3a^2y)^2 - a^6$ by $x^2 + y^2 - a^2$.

Establish the identities—

$$\begin{aligned} & (a^2 + b^2 + c^2 + bc + ca + ab)(bc + ca + ab) \\ & \quad - 2(a + b + c)abc \\ &= (b^2 + ca)(c^2 + ab) + (c^2 + ab)(a^2 + bc) \\ & \quad + (a^2 + bc)(b^2 + ca), \\ & (x^2 + 2yz)^3 + (y^2 + 2zx)^3 + (z^2 + 2xy)^3 \\ & \quad - 3(x^2 + 2yz)(y^2 + 2zx)(z^2 + 2xy) \\ &= (x^3 + y^3 + z^3 - 3xyz)^2. \end{aligned}$$

3. If α, β be the roots of $ax^2 + 2bx + c = 0$, prove that $ax^2 + 2bx + c = a(x - \alpha)(x - \beta)$.

Solve the equations—

$$(1) a(b - c)x^2 + b(c - a)x + c(a - b) = 0.$$

$$(2) \frac{x^2}{a} + \frac{y^2}{b} = \frac{a^2}{x} + \frac{b^2}{y} = a + b.$$

4. If $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \dots = \frac{a_n}{b_n}$ then each fraction is equal to

$$\left\{ \frac{p_1 a_1^m + p_2 a_2^m + \dots + p_n a_n^m}{p_1 b_1^m + p_2 b_2^m + \dots + p_n b_n^m} \right\}^{\frac{1}{m}}$$

$$\text{If } \frac{P}{pa^2 + 2qab + rb^2} =$$

$$\frac{Q}{pac + q(bc - a^2) - rab} = \frac{R}{pc^2 - 2qca + ra^2}$$

prove that P, p, Q, q , and R, r may be interchanged without altering the equalities.

5. Write down the general term of the expansion of $(1 - x)^{\frac{p}{q}}$ in powers of x .

If x be small compared with N^2 , prove that $\sqrt{N^2 + x}$ is approximately equal to $N + \frac{x}{4N} + \frac{Nx}{2(2N^2 + x)}$ and shew that the error is of the order $\frac{x^2}{N^7}$.

Ex.—Shew that $\sqrt{101} = 10.0498756$ to eight places of decimals.

6. Find the number of combinations of n things taken r together, without assuming the formula for permutations.

A man goes in for an examination in

which there are four papers, with a maximum of m marks for each paper. Shew that the number of ways of getting half marks on the whole is

$$\frac{1}{2}(m+1)(2m^2 + 4m + 3).$$

7. Explain, and state the several advantages of, the chief systems of angular measurement in use.

Prove that the circumferences of circles vary as their radii; and mention the approximations to their constant ratio which are practically employed.

Shew that there are eleven pairs of regular polygons which satisfy the condition that the measure of an angle of one in degrees is equal to the measure of an angle of the other in grades; and find the number of sides in each.

8. Define the sine of an angle; and find the value of the sines of angles of $135^\circ, 240^\circ, 292\frac{1}{2}^\circ, 432^\circ$.

Shew that, $\sin^2 10^\circ + \cos^2 20^\circ - \sin 10^\circ \cos 20^\circ = \sin^2 10^\circ + \cos^2 40^\circ + \sin 10^\circ \cos 40^\circ = \frac{3}{4}$.

9. Prove geometrically that—

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}.$$

Solve the equation—

$$\cos x + \sin 3x + \cos 5x + \sin 7x + \dots + \sin (4n-1)x = \frac{1}{2}(\sec x + \operatorname{cosec} x).$$

10. Find an expression for $\cos(x_1 + x_2 + x_3)$ in terms of sines and cosines of x_1, x_2, x_3 .

State the corresponding theorem for the case of n angles $x_1, x_2, \dots, x_n$.

If $\cos(y-z) + \cos(z-x) + \cos(x-y) = -\frac{3}{2}$, shew that

$$\begin{aligned} & \cos^3(x+\theta) + \cos^3(y+\theta) + \cos^3(z+\theta) \\ & \quad - 3 \cos(x+\theta) \cos(y+\theta) \cos(z+\theta) \end{aligned}$$

vanishes whatever be the value of θ .

11. Shew how to solve a triangle, having given the three sides, proving from the formulæ obtained that there cannot be more than one triangle, though there may be none, with the given parts.

The perpendiculars from the angular points of an acute-angled triangle ABC on the opposite sides meet in P , and PA, PB, PC are taken for the sides of a new triangle. Find