where $g$, $k$, $a$, $c$, $h$, $\theta$ and $\phi$ are rational; and $z = e^2 + 1$, $e$ being rational. It is readily seen that

$$g = -\frac{p_2}{10}, \text{ and } k = -\frac{p_3}{20}.$$

Because $u_1^5 = \frac{(u_1^2 u_3)^2 (u_2^2 u_1)}{(u_2 u_3)^2}$, it follows from (2) and (3) that

$$\left.\begin{aligned} u_1^5 &= B + B'\sqrt{z} + (B'' + B'''\sqrt{z})\sqrt{(hz + h\sqrt{z})} \\ u_4^5 &= B + B'\sqrt{z} - (B'' + B'''\sqrt{z})\sqrt{(hz + h\sqrt{z})} \\ u_2^5 &= B - B'\sqrt{z} + (B'' - B'''\sqrt{z})\sqrt{(hz - h\sqrt{z})} \\ u_3^5 &= B - B'\sqrt{z} - (B'' - B'''\sqrt{z})\sqrt{(hz - h\sqrt{z})} \end{aligned}\right\} \tag{4}$$

where $B$, $B'$, $B''$ and $B'''$ are rational functions of $a$, $c$, $e$, $h$, $\theta$ and $\phi$. In like manner, because $u_1^3 u_2 = \frac{(u_1^2 u_3)(u_2^2 u_1)}{u_2 u_3}$, we have from (2) and (3)

$$u_1^3 u_2 = A + A'\sqrt{z} + (A'' + A'''\sqrt{z})\sqrt{(hz + h\sqrt{z})},$$

where $A$, $A'$, $A''$ and $A'''$ are rational. The value of $A$ is

$$A = \frac{1}{g^2 - a^2 z}\{g(k^2 - c^2 z) + azhe(\theta^2 - \phi^2 z)\}. \tag{5}$$

From these data, the six equations, involving the six unknown quantities $a$, $c$, $e$, $h$, $\theta$ and $\phi$, are (see *Journal of Mathematics* as above) obtained:

$$\left.\begin{aligned} p_4 &= -20A + 5g^2 + 15a^2 z \\ p_5 &= -4B + 40acz \\ B'' &= 1 \\ B''' &= 0 \\ hz(\theta^2 + \phi^2 z + 2\theta\phi) &= k^2 + c^2 z - g(g^2 - a^2 z) \\ h(\theta^2 + \phi^2 z + 2\theta\phi z) &= 2kc - a(g^2 - a^2 z) \end{aligned}\right\} \tag{6}$$

Our business is to obtain $u_1^5$, $u_2^5$, $u_3^5$ and $u_4^5$ from these equations.

§3. It will be found that $a^2 z$ is the root of an equation $F(y) = 0$, whose coefficients are rational functions of $p_2$, $p_3$, $p_4$ and $p_5$, and which, when $p_2$ is zero, is of the sixth degree. Since $a^2 z$ is rational, it follows that, when the coefficients of the given quintic are commensurable, the equation $F(y) = 0$ has a commensurable root. Let this be found. Then $a^2 z$ is known. The formulæ from which the equation $F(y) = 0$ is obtained give us, along with $a^2 z$, the value of $\frac{c}{a}$. The remaining elements necessary for the determination of $u_1^5$, $u_2^5$, $u_3^5$ and $u_4^5$ may then be obtained from linear equations, without finding