

set to the middle latitude  $3^{\circ} 25'$ , and the departure 115 taken on A, opposite it on index will be seen the difference of longitude 116 miles.

Long. left.....  $49^{\circ} 32' W.$   
Diff. long. 116 miles..... =  $6^{\circ} 50' E.$   
Long. in.....  $12^{\circ} 36' W.$

Hence her course is S.  $31^{\circ} 4' E.$ , distance run 804.2, and longitude in  $12^{\circ} 36' W.$

A ship from latitude  $46^{\circ} 35' N.$ , and longitude  $176^{\circ} 42' W.$ , sails N. W. by W.  $\frac{1}{2} W.$  till she arrives in latitude  $51^{\circ} 18' N.$ ; required the distance run and longitude in.

Lat. left.....  $46^{\circ} 35' N.$ .....  $46^{\circ} 35' N.$   
Lat. in.....  $51^{\circ} 18' N.$ .....  $51^{\circ} 38' N.$   
Diff. lat.....  $4^{\circ} 43'$ .....  $9^{\circ} 7' 53''$   
60  
Diff. in miles 283  
Mid. lat  $48^{\circ} 56'$

This problem can be solved by a process nearly similar to that made use of in the third example; yet, in order to show the powers of the scale, we shall adopt a different method. It may, however, be proper to remark, before beginning, that when the larger divisions are considered degrees, each of the smaller ones will represent six minutes, being the tenth part of sixty.

**RULE.**—Set the index to middle latitude  $48^{\circ} 56'$ , and on side A take the difference of latitude  $4^{\circ} 43'$ , that is, 4 large divisions and 7 1-6 small ones, or the division representing 47.16; opposite this will be found on index the meridional difference of latitude  $7.2$  or  $7^{\circ} 12'$ .

Now, set the index to the course  $51\frac{1}{2}$  points, and opposite the difference of latitude  $4^{\circ} 43'$  on A, will appear on index the distance  $10^{\circ}$  or 600 miles; and if half \* the meridional difference of latitude  $3^{\circ} 36'$ , or the division representing 36, be taken on A and its perpendicular traced to index, then opposite this point on index will be found on B half the difference of longitude  $6^{\circ} 72'$  or  $6^{\circ} 41'$ . Hence the difference of longitude is  $13^{\circ} 28'$ , and longitude in  $169^{\circ} 50' E.$

The object of this work being to teach the application of the scale to Navigational purposes, and not to throw any additional light on Navigation, it is not thought necessary to treat on oblique and current sailings here. If the operator thoroughly understands Trigonometry and its application to Traverse Sailing, any cases that may occur in these, however, will not cost him a moment's thought when in possession of the scale.

### SPHERICAL TRIGONOMETRY.

65. In treating on Spherical Trigonometry at all, our object is merely to show that the scale is adapted as well to Spherical as to Plane Trigonometry. We shall therefore give only a few examples.

**Ex. 1.**—In the spherical triangle ABC, right-angled at B, the hypotenuse AC is  $64^{\circ}$ , and the angle C  $46^{\circ}$ ; find B C.

To find B C:

Cot. A C  $64^{\circ}$ : Rad. = Cosine C  $46^{\circ}$ : Tang. B C.

The cot. of  $64^{\circ}$  (Art. 21) is 29.3, radius is 60, and cosine  $46^{\circ}$  is 43.6. Set radius 60 on index to 29.3 on A, then opposite 43.6 on A is tangent B C 85 on index; take 85 on line of tangents, set the index to it, and on quadrant will appear the number of degrees in the arc B C  $54^{\circ} 55'$ .

**Ex. 2.**—Let the hypotenuse AC and side BC of the figure AB (Ex. 1) be given, equal to  $70^{\circ} 24'$  and  $65^{\circ} 10'$  respectively: find angle C.

\* Because the perpendicular of the meridional difference of latitude will not intersect the index, its half is used.

Rad.: Cot. A C  $70^{\circ} 24'$  = Tang. B C  $65^{\circ} 10'$ : Cos. C.

Radius is 60, cotangent  $70^{\circ} 24'$  is 21.2, and the semi-tangent of  $65^{\circ} 10'$  is 65; then, set 60 on index to 21.2 on B, and opposite 65 on index is half the cosine C 23.1 on B; therefore cosine angle C is 46.2. The perpendicular of 46.2 taken on A, being traced to the arc of the quadrant, will indicate on it the number of degrees  $39^{\circ} 42'$ .

### ASTRONOMICAL PROBLEMS.

66. To find the sun's longitude on a given day.

**RULE.**—Count the number of days from the nearest equinoctial point; and if the sun is on the south side of the equator, their number will very nearly agree with the sun's longitude taken in degrees on the quadrant of the scale. If the declination be north, count the number of days as before, and subtract one day for every thirty, and in proportion for a less number, and the remainder will agree with the sun's longitude in degrees and minutes on the quadrant.

**Note.**—The sun's longitude is often useful to discover data for the solution of problems in Astronomy.

**Ex. 1.**—Required the sun's longitude on the 25th day of November, 1860.

The number of days from the 22d September (the day on which the sun was on the equator) to the 25th day of November, is 64; hence the sun's longitude on that day was  $64^{\circ}$ .

**Ex. 2.**—Required the sun's longitude on the 25th day of May, 1860.

From the 20th March (the day on which the sun was on the equator) to the 25th May, are 66 days; and subtracting a day for every 30, that is 2 1-5 or 2.2 days, leaves 63.8 or  $63^{\circ} 48'$ , the sun's longitude.

67. To find the sun's declination on a given day.

**Ex.**—Required the sun's declination on the 25th day of November.

The sun's longitude by (Art. 66) is  $61^{\circ}$ .

Then, as radius..... = 60 on F  
Is to sine  $61^{\circ}$  (the sun's decl.)..... = 51 on B  
So is sine of  $23^{\circ} 28'$  (greatest decl.)..... = 21 on F

To sine present decl.  $20^{\circ} 55'$ ..... 21.36 on B

68. The greatest declination and the present declination given to find the sun's longitude.

**Ex.**—Given the greatest declination  $23^{\circ} 28'$ , and the present declination  $20^{\circ} 55'$ : to find the sun's longitude.

**RULE.**—As sine of  $23^{\circ} 28'$  (greatest decl.) 21 on F  
Is to sine  $20^{\circ} 55'$  (present decl.)..... 21.36 on B  
So is radius..... 60 on F

To sine of sun's longitude  $61^{\circ}$ ..... 54 on B

69. The latitude and declination given to find the sun's amplitude, or the distance in degrees the sun is from the east or west at its rising or setting.

**Ex.**—Given the latitude  $40^{\circ} N.$  and the declination  $22^{\circ} 30' N.$ : required the sun's amplitude at rising.

**RULE.**—As cosine lat.  $40^{\circ}$ ..... 46 on F  
Is to sine decl.  $22^{\circ} 30'$ ..... 23 on B  
So is radius..... 60 on F

To sine amplitude  $29^{\circ} 50'$  nearly..... 29.8 on B

70. To find the time of the sun's rising and setting on a given day in any latitude.

**Note.**—If the declination is not given, find it by Art. 67.

**Ex. 1.**—Required the time of the sun's rising and setting in latitude  $50^{\circ} N.$ , declination being  $23^{\circ} 88' N.$

As radius..... 60 on B  
Is to tang. lat.  $50^{\circ}$ ..... 71.2 on F  
So is tang. decl.  $23^{\circ} 28'$ ..... 26 on B

To sine ascensional difference  $31^{\circ}$  = 33.1 on F

The ascensional difference converted into time (allowing  $15^{\circ}$  to hour and  $1^{\circ}$  to 4 minutes of time), gives the time that the sun rises before, or sets after, 6 o'clock in summer, and the reverse in winter, in north latitude. The above ascensional difference  $31^{\circ}$ , converted into time, gives 2 hours 4 minutes, which being added to 6 o'clock, gives the time of the sun's setting 8 hours 4 minutes, and being subtracted from 6 o'clock gives 3 hours 56 minutes; therefore the sun sets at 4 minutes past 8 and rises at 56 minutes past 3.

**Ex. 2.**—Required the time of the sun's rising in lat.  $40^{\circ} N.$ , the declination being  $15^{\circ} N.$

As radius..... 60 on F  
Is to tang. lat.  $40^{\circ}$ ..... 50.2 on B  
So is tang. decl.  $15^{\circ}$ ..... 16.2 on F

To sine ascensional difference  $13^{\circ}$ ..... 13.6 on B

13 degrees converted into time gives 52 minutes, which, being subtracted from 6 o'clock, gives 5 hours 8 minutes; hence the sun rises at 8 minutes past 5 o'clock.

71. To find the length of the longest day in any latitude under  $66^{\circ} 32'$ .

The longest day will happen when the sun is in the solstice, at which time the declination is  $23^{\circ} 28'$ .

**Ex.**—Required the longest day in latitude  $58^{\circ}$ .

As radius..... 60 on B  
Is to tang. lat.  $58^{\circ}$ ..... 95 on F  
So is tan. of decl.  $23^{\circ} 28'$ ..... 26 on B

To sine ascensional difference  $43^{\circ}$  = 41 on F

43 degrees converted into time is equal to 2 hours 52 minutes, and this added to 6 o'clock (Art. 70) gives the time of the sun's setting 8 hours 52 minutes, which, being doubled, gives the length of the day 17 hours 44 minutes.

72. To find the length of the longest day in any latitude above  $66^{\circ} 32'$ .

**Ex.**—What is the length of the longest day at the North Cape, in the Island of Myræroe, in latitude  $71^{\circ} 30' N.$ ?

**RULE.**—Set the index to lat.  $71^{\circ} 30'$  on quadrant, and on the perpendicular of 30 taken on A will be found the semi-tangent of the latitude 89.3.

Then take 89.1 on index and set it to the parallel of half radius 30 on B, and opposite 60 (sine of ascensional difference for 6 hours) on index will be found on B 20.2, the tangent of declination on the day on which the sun ceases to set in the given latitude. Set the index to 20.2 on the line of tangents, and the declination will appear on the arc to be  $18^{\circ} 35'$ , and its sine will be found on side B to be 19.1.

Set 21 (sine of greatest declination  $23^{\circ} 28'$ ) on index to 39.1 (sine of aforesaid decl.) on B, and on the arc of the quadrant will appear the sun's longitude when it ceases to set  $53^{\circ} 35'$ . Subtract  $51^{\circ} 35'$  from 90, and the remainder  $38^{\circ} 25'$  doubled gives  $76^{\circ} 50'$ , which, being taken in time, is equal to 76 days 20 hours.

The operation may be more easily understood by the following proportions:—

As semi-tangent lat.  $71^{\circ} 30'$ ..... 89.1 on index  
Is to half radius..... 30 on B  
So is sine ascensional diff. for 6 hours..... 60 on index

To tangent decl. when the sun ceases to

set in the given latitude  $18^{\circ} 35'$ ..... 20.2 on B

The sine of  $38^{\circ} 35'$  is equal to..... 19.1

Then, as sine of greatest decl.  $23^{\circ} 28'$ ..... 24 on F

Is to sine of above decl.  $18^{\circ} 35'$ ..... 19.1 on B

So is radius..... 60 on F

Sine of sun's long.  $51^{\circ} 35'$ ..... 47 + on B