SOLUTION FIFTH.

PROPOSITION 7.

THEOREM.

In the same manner as demonstrated, (Lem. 10,) the arc Kk″ is the variation of the point K′ towards o′, and oq″ is the variation of o′ towards q′, and Ky″ is the variation of K towards o, and op″ is the variation of o towards q, and that K′*x*″ and K′y″, will each become the variation kk″—that o′q″ and op″ will become the variation *ii*″ (Lem. 10.)—Then from the point B as a center, with the distances Bk″ and Bi″, and from D′ as a center, with the distances D*e*′ and D′*f*, describe the intersections a′ and b′, and through the points a′ and b′, draw the straight line a′b′n meeting the circular arc BD′ in the point n,—the distance An shall be equal to the length of the circumference of the circle ABD, (Fig. 4.) FIG. 8.

For it is evident, (Lem. 3), that n is the ultimate intersection of all the intersections described through the series of points k″ and *i*″ &c. and through the series of points *e* and *f* &c., and that the curve line a′b′ ... n must be on the same straight line with its chord a′n. (prop. 1) ; consequently the point n must be on the intersection of the arcs *n*N and BD′—and the distance An equal to A*n* (Fig. 5.)—and equal to the circumference of the circle ABD, (Fig. 4.)

SOLUTION SIXTH.

PROPOSITION 8.

THEOREM.

The straight line abn drawn through the points of intersections a and b, and the straight line a′b′n drawn through the points a′ and b′, intersecting each other in the point n—The distance An shall be the determinate length of the circumference of the Circle ABD, Fig. 4. FIG. 8.

The demonstration for this Solution is the same as that of Solution Third.

Case Third.—*When the arc Bdef ... hikD′, is the arc of a circle of the radius AC, described through the points B and D′, intersecting the series of arc 1″K′ only,* (*Lem. Fig.* 3, 5.)

SOLUTION SEVENTH.

PROPOSITION 9.

THEOREM.

Make the arc B1″, B2″, B3″, &c., and B2′, B3′, B4′, &c., on the arc BC, equal to the arcs B1, B2, B3, &c., and B2′, B3′, B4′, &c., of the arc BD, of Fig. 5—and from C as a center with the distances C1″, C2″, C3″, &c., and C2′, C3′, B4′, &c., describe the arcs 1″T′, 2″S′, 3″R′, &c., and 2′K′, 3′O′, 4′Q′, meeting the arc CD in the points T′, S′, R′, &c., and K′, O′, Q′. Then bisect the arc CD in the point D′, and with AC as a radius, through the points B and D′ describe the arc BD′, intersecting the arc 1″T′ in the point d—the arc 2″S′ in the point *e*—the arc 3″R′ in the point *f*, &c.—and the arc 2′K′ in the point k—the arc 3′O′ in the point *i*—and the arc 4′Q′ in the point h.—Next from the point A as a center, with the distances Ad, A*e*, A*f*, &c., and Ak, A*i*, and Ah, &c,, describe the arcs 1T, 2S, 3R, and the arcs 2′K, 3′O, and 4′Q, meeting the arc BD in the points 1, 2, 3, &c., and 2′, 3′, 4′, &c., and meeting the arc CD in the points T, S, R, &c., and K, O, and Q, &c. ; also through the points B and K′, and B and K, with AC for a radius, describe the arcs BK′, and BK.—Again from A as a center with the distances At, Au, describe the arcs tu′ and uv′—and from K′ as a center with the distances K′u′ and K′v′ describe the arcs u′u″ and v′v″ ; also from C as a center with the distances Cs and Cr describe the arcs ss′ and rr′, and from K as a center with the distances Kr′ and Ks′, describe the arcs r′r″ and s′s″. The arc tu″ is the variation of t towards u, and the arc uv″ is the variation of u towards v on the arc tK′ ; also the arc t′s″ is the variation of t′ towards s, and the arc sr″ is the variation of s towards r, on the arc t′K—and from D′ as a center with the distances D′*e*″ and D′*f*″ describe the arcs *e*′*e*″ and *f*′*f*″—the arc d*e*″ is the variation of d towards *e*, and *ef*″ is the variation of *e* towards *f* on the arc dk, (Lem. 9.) The demonstration of this is the same as for Proposition 5. FIG. 9.