Prove that a quadratic equation can have only two roots.

5. Solve the equations

(1) $\sqrt{2x}+\sqrt{3x}=\sqrt{5}$. Ans $25-10\sqrt{6}$.

(2) $\left\{(x+l)^2-a^2\right\}\left\{(x+l)^2-b^2\right\}=\left\{(x+m)^2-a^2\right\}\left\{(x+m)^2-b^2\right\}$

Sol. $(x+l)^4-(a^2+b^2)(x+l)^2=(x+m)^4-(a^2+b^2)(x+m)^2$

$\therefore \left\{(x+l)^2-(x+m)^2\right\}\left\{(x+l)^2+(x+m)^2\right\}-(a^2+b^2)\left\{(x+l)^2-(x+m)^2\right\}=0.$

$\therefore (x+l)^2+(x+m)^2-(a^2+b^2)=0 \quad (1)$

and $(x+l)^2-(x+m)^2=0 \quad (2)$

$\therefore$ from (1) $x=\dfrac{-(l+m)\pm\sqrt{2lm+b^2+a^2}}{2}$

from (2) $x=-\frac{1}{2}(l+m)$

(3) $\dfrac{1}{x-3}+\dfrac{3}{x+15}+\dfrac{1}{x+3}-\dfrac{5}{x+9}=0$

Sol. $\dfrac{4x+6}{x^2+12x-45}-\dfrac{4x+6}{x^2+12x+27}=0$

$\therefore \dfrac{1}{x^2+12x-45}-\dfrac{1}{x^2+12x+27}=0$

$\therefore x=\infty$

and $4x+6=0$; $\therefore x=-\frac{3}{2}$

(4) $\left.\begin{array}{l}(1)\ \dfrac{1}{x}+\dfrac{1}{z}=\dfrac{2}{y}\\ (2)\ x+z=\dfrac{1}{4y}\\ (3)\ x^2-2yz=\dfrac{1}{12}\end{array}\right\}$

Sol.—From (1) $\dfrac{x+z}{xz}=\dfrac{2}{y}$

$\therefore$ combining this with (2)

$$xz=\frac{1}{8}$$

and substituting in (2)

$$y=\frac{2x}{8x^2+1}$$

And substituting both of these last results in (3) and multiplying up.

$96x^4+4x^2-7=0 \therefore x^2=-\frac{7}{24}$ or $\frac{1}{4}$

from which, discarding the first value, we find $z=\pm\frac{1}{2}$ and $y=\pm\frac{1}{3}$.

6. Find the sum of n terms of an arithmetical series, having given the first term and the com. diff.

Find the sum of 32 terms of an A. P. whose 5th term is 20, and whose 21st term is 15.

Sol.—Let a be 1st term and b com. diff., then $20=a+4b$; $15=a+20b$, $\therefore b=-\frac{5}{16}$ and $a=21\frac{1}{4}$

$\therefore$ sum of 32 terms is 525.

7. Define an harmonic series, and shew how to insert m harmonic means between a and b.

If a, $2b$ and c be in H. P., then will $a+c$, a and $a-b$ be in G. P., and also will $a+c$, c, $c-b$.

Sol.—Since a, $2b$, c are in H. P., then

$b=\dfrac{ac}{a+c}$ $\therefore$ (1) $ac-ba-bc+a^2=a^2$

$\therefore \sqrt{(a+c)(a-b)}=a$

$\therefore$ (2) $ac-ba-bc+c^2=c^2$

$\therefore \sqrt{(a+c)(c-b)}=c$

8. Find the No. of combinations of n things taken v at a time, and prove that it is the same as the No. of combs. of n things taken $n-v$ at a time.

Prove that the No. of combs. of $2n$ things taken n at a time is

$$2n\frac{1.3.5\ldots(2n-1)}{1.2.3\ldots n}$$

Sol.—The No. of combs. of $2n$ things taken n at a time is

$$\frac{2n(2n-1)\ldots\ldots(n+1)}{1.2.3\ldots n}$$

that is $\dfrac{2n,\ 2n-1,\ 2(n-1)\ldots\ldots 4.3.2.1.}{1.2.3\ldots n\times 1.2.3\ldots n}$

that is, taking every second term in the numerator,

$$\frac{1.3.5\ldots\ldots(2n-1)\times 2^n\times 1.2.3\ldots n}{1.2.3\ldots n\times 1.2.3\ldots n}$$

9. Assuming the truth of the Binomial Theorem when the index is a whole number, prove it when the index is a positive fraction.

Write down the fifth term of $(2^3-2)^{-n}$

Prove that

$$\sqrt[3]{\tfrac{1}{6}}=\tfrac{1}{2}+\frac{1}{3.2^2}+\frac{1.4}{1.2}\frac{1}{3^2 2^5}$$