ECONOMICAL SIZE OF PIPE FOR GIVEN LOSS OF HEAD*

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THE most economical size of penstock is a matter of the greatest importance when the penstock is a long pipe under high head. In such a case the first cost of the penstock is apt to be the largest part of the total first cost of the plant and it is therefore very important that the most economical size of the penstock be determined.

Before the most economical size of penstock can be determined, it is necessary to know what principal of economy is to be used. This principal may be different for different plants. For instance, it may be required to determine the size of the pipe so that the total net annual income shall be a maximum, or so that the percentage return on the original investment shall be a maximum. In other cases it may be required to determine the pipe so that the percentage return on the original investment shall be a specified amount. This may be the case when the amount of water available is limited and when it is imperative to produce as much power as possible, consistent with a fair rate of return on the original investment. Thus, the economic design of a penstock may come up in a variety of forms.

When the penstock is to consist of a pipe of constant diameter throughout, the problem, so far as the penstock is concerned, is comparatively simple. Assume, for instance, that the net annual income shall be a maximum. A simple and direct method of procedure in determining the economical size of pipe of constant diameter throughout, under the given principle of economy, is as follows:—

Calculate or approximate the total net annual income for each of several assumed diameters. With the diameters as abscissae and the total net annual income as ordinates, a curve may be plotted. From this curve, the diameter can be found for which the total net annual income will be a maximum.

In the case of high heads, however, it is often found economical to make the pipe consist of two or more sections of different diameters. Such a pipe would approximate the shape of a funnel with the smallest diameter where the head is the highest. In other cases it is found economical to make the upper part—the part under the low head—consist of concrete or staves and the remainder of steel. The determination of the economical pipe now reduces to the determination of the diameters of the several sections and the problem may not be an easy one. The method of procedure suggested for the case of a pipe of constant diameter throughout, is no longer applicable. While considerable latitude is left to the designer, it is nevertheless important to be able to determine what the theoretically economical diameters of the several sections are.

In 1907, A. L. Adams proposed a principle for the design of a penstock that seemed to him self-evident. This principle may be stated as follows:—

That pipe is most economically designed for which the interest and depreciation on first cost of the pipe plus the annual value of the power lost due to friction in the pipe is a minimum.

This principle of economy seems simple and logical and it is referred to in nearly every discussion on the economical size of penstock. Frequently, formulas for the economical diameter are derived from it. For the case of riveted steel, Professor D. W. Mead (Mead's Water Power Engineering, 2d edition, page 546) derives two formulas:—

(1) $d = 0.2153 \sqrt[7]{\dfrac{fbq^3 S}{cih}}$ for high head

and

(2) $d = 0.2195 \sqrt[6]{\dfrac{fbq^3}{t'ci}}$ for low head

where h is the head on the point under consideration; f is the coefficient of friction defined in equation (3) which follows; S is the allowable unit stress in shell in pounds per square inch corrected for rivet holes and water hammer; c is the cost of the metal per pound; i is the rate of interest and depreciation on first cost of pipe; b is the annual value of a horsepower at the wheel; and q is the discharge in cubic feet per second.

When the required theoretical thickness of the shell is more than the minimum allowable thickness, the pipe is considered to be designed for high head and formula (1) applies. When the theoretical thickness of the shell becomes less than the minimum allowable thickness, formula (2) applies, which was derived on the asumption that the thickness of the shell shall be the minimum allowable thickness, t'.

From formula (1) it is seen that the theoretically economical diameter of a pipe under high head decreases as the head h, increases; and from (2) it is seen that the economical diameter for a pipe under low head is independent of h.

In deriving formulas (1) and (2) the friction head was assumed to be given by

(3) $h' = \dfrac{fl}{d} \cdot \dfrac{v^2}{2g}.$

Now f is unknown to start with since it depends upon the diameter of the pipe which is not yet determined. In practice, a value of f is first assumed, and the diameters of the pipe are determined by means of the above formulas. If the average value of f for the pipe so determined differs appreciably from that assumed, a revised value for f is taken.

If the friction head for riveted steel pipes is assumed to be given by the formula

(4) $h = 0.00050 \dfrac{v^2 l}{d^{1.25}}$;

the resulting formulas become:

(5) $d = 0.1423 \sqrt[7.25]{\dfrac{bq^3 S}{cih}}$ for high head;

and

(6) $d = 0.1356 \sqrt[6.25]{\dfrac{bq^3}{t'ci}}$ for low head.

Formulas (5) and (6) do not contain the unknown coefficient f. So far as the writer knows, formulas (5) and (6) are here given for the first time.

If the value of b, the value of a hydraulic horsepower at the wheel, is given, Adams' principle and the resulting formulas will solve the problem. Unfortunately the proper value for b is not always known. As already stated the principle of economy to be used in the design of a penstock may be one of several, and Adams' principle may be difficult to apply. Engineers of experience declare that this principle is not always applicable. These considerations urged the writer to see if a method of attack analogous to that suggested for a pipe of constant diameter throughout could be used, and as a result he has arrived at certain conclusions that may be of interest to students of hydraulics.

If for an assumed total loss of head h', the most economical pipe could be found, the following method would be applicable: For each of several assumed values for the total loss of head, h', in the pipe, determine the economical pipe, and if, the total percentage return, p, is to be a specified amount, calculate or approximate the *percentage return* for each assumed value of h'. A curve can then be drawn with h' as the abscissa and p as the ordinate. From this curve the value of h' can be obtained so that the percentage return shall be the desired amount. Now with h' determined, the most economical pipe can be found.

If the pipe is homogeneous—made entirely of riveted steel for instance—the pipe of least annual cost will be the pipe of least first cost, or of least amount of metal. If the pipe is not homogeneous, such as when the upper part is of staves and the lower part of steel, it is no longer a question of least amount of metal but rather one of least first cost or least annual cost. As shown later, the expression that will determine the diameter of the pipe for least annual cost will also determine the pipe for least first cost or least amount of metal (pipe homogeneous) if one or two of the

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