

same as (138)] "celles qui satisfont au problème proposé doivent avoir la propriété nonseulement que les fonctions symétriques de R_1, R_2 , etc., soient rationnelles en A, B, C , etc. (ce qu'Abel a remarqué), mais aussi que les fonctions cycliques des quantités R_1, R_2 , etc., prises dans un certain ordre, soient également rationnelles en A, B, C , etc.; en d'autres termes, l'équation de degré $\mu - 1$, dont R_1, R_2 , etc., sont les racines, doit être une équation abélienne. J'entendrai toujours ici par équations abéliennes cette classe particulière d'équations résoluble qu'Abel a considérées dans le Memoire XI du premier volume des *Œuvres complètes*, et dont je supposerai les coefficients fonctions rationnelles de A, B, C , etc. En désignant par $x_1, x_2, \dots, x_n$, des racines prises dans un ordre déterminé, ces équations peuvent être définies soit en disant que les fonctions cycliques des racines sont rationnelles en A, B, C , etc., soit en disant qu'on a les relations,

$$x_2 = \theta(x_1), x_3 = \theta(x_2), \dots, x_n = \theta(x_{n-1}), x_1 = \theta x_n,$$

où $\theta(x)$ est une fonction entière de x dont les coefficients sont rationnels en A, B, C , etc." In saying that the $\mu - 1$ (or, in our notation, the $n - 1$) terms, R_1, R_2 , etc., are the roots of an Abelian equation, Kronecker must be understood to assume that the equation $\phi(x) = 0$, which has the terms in (139) for its roots, is irreducible. As a matter of fact, in the most general case, which includes all the others, the equation $\phi(x) = 0$ is irreducible. But in particular cases it may be reducible, and then it is not an Abelian. In a paper by the present writer, entitled "Principles of the Solution of Equations of the Higher Degrees," which appeared in this Journal (Vol. VI, No. 1), it was proved that when the equation $\phi(x) = 0$ is reducible, it can be broken into a number of irreducible equations,

$$\psi_1(x) = 0, \psi_2(x) = 0, \dots, \psi_s(x) = 0,$$

each a pure uni-serial Abelian. Hence, for a detailed discussion of the problem we have now before us, we should require to deal not only with the general case in which the equation $\phi(x) = 0$ is irreducible, but also with the several cases in which equations such as $\psi_1(x) = 0, \psi_2(x) = 0$, etc., can be formed. But since, as has been stated above, the particular cases are included in the general, we shall confine ourselves to the problem of the necessary and sufficient forms of the roots of the solvable irreducible equation $f(x) = 0$ of degree n , when the subordinate equation $\phi(x) = 0$ of degree $n - 1$ is irreducible, and is therefore a pure uni-serial Abelian; it being understood that $n - 1$ is either the continued product of a number of distinct primes, or four times the continued product of a number of distinct odd primes.