

Again, taking for an example any quantity, 27,132, being the 20th term of Column VII. This quantity is also found to occupy the place of the 20th term in Column XIV. It can, therefore, be represented by the following series :

$$\begin{aligned} 27,132 &= 18,564 + 8,568 \\ &= 8,568 + 18,564 \end{aligned}$$

27,132 is equal to the sum of the following series :

| | |
|--------|--------|
| 1 | 1 |
| 6 | 13 |
| 21 | 91 |
| 56 | 455 |
| 126 | 1820 |
| 252 | 6188 |
| 462 | 18564 |
| 792 | |
| 1287 | 27,132 |
| 2002 | |
| 3003 | |
| 4368 | |
| 6188 | |
| 8568 | |
| <hr/> | |
| 27,132 | |

It follows from these properties, that if any one of the 20 horizontal columns be moved one square to the left, the figures in each square are the sum of the entire series above it. This holds good for any number of vertical and horizontal columns, 20, 40, 100 or 1000.

It will be observed that every one of the quantities given in the above series is also the sum of a series preceding it. For instance the quantity—

| | |
|------|---|
| 8568 | is the sum of the series in Column V, beginning at 2380 |
| 6188 | " " " " 1820 |
| 4368 | " " " " 1365 |
| 3003 | " " " " 1001 |

And so on to the top of the column.

And this character holds good for each and all the figures in Bernoulli's Table. Each and all after unity are sums of preceding series of figures given in the table. Hence the applicability of Bernoulli's legend, the conception of which he derived from the Logarithmic Spiral and applied to himself—

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If figures in any square be selected, such as 19,448, being in the 18th horizontal and VIIIth vertical column, then the sum of the figures covered by continuous movement one square to the left and one square upwards, always to the left and to the extremity of the Board, will be equal to the quantity in the second square below the square from which the start was made—less unity. But if the square occupied by the figures denoting the number of the horizontal column be occupied by cyphers and one step more be made the sum will be equal.

Example. Starting from 19,448, being the 18th term in the VIIIth column, the Series is—

| | |
|--------|---------------------------------|
| 19,448 | 6,188 |
| 12,376 | 2,380 |
| 8,008 | 1,820 |
| 4,368 | 560 |
| 3,003 | 455 |
| 1,365 | 105 |
| 1,001 | 91 |
| 364 | 14 |
| 286 | 13 |
| 78 | 1 |
| 66 | 1 |
| 12 | |
| 11 | 11,628 the 20th term column VI. |
| 1 | |
| <hr/> | |
| 50,387 | |

The 20th term in the VIIIth column is 50,388.

Numerous other properties are pointed out by Bernoulli, and mathematically proved. Also in Francis Maseres translation many curious features are noticed and subjected to mathematical analyses.

At the close of Chapter VI, I have introduced a formula which brings Bernoulli's formula and Table within the range of any one familiar with the elements of algebra. But it is the INTERCHANGEABLE property possessed by the quantities which gives them present importance.

The following Formula is derived from Bernoulli's 12th Property. It develops some remarkable relations, and is especially useful for obtaining any desirable ratio or approximation to that ratio in the form of two series of numbers—

The application of the letters is given in Table I.

| | | |
|-----------|-------|------------------------|
| S | = | $\frac{1 \times n}{n}$ |
| Therefore | S × a | = 1 × n |
| And | S : n | = 1 : a |