# Mathematical Department.

Communications intended for this part of the JOURNAL should be on separate sheets, written on one side only, and properly paged to prevent mistakes. They must be received on or before the 20th of the month to secure notice in the succeeding issue, and must be accompanied by the correspondents' names and addresses.

## ALGEBRAIC EXERCISES.

[SELECTED.]

### I.

Examples such as the following afford possibly the best elementary exercise that has yet been devised for cultivating facility in manipulating algebraic expressions.

1. If $s=a+b+c+d$, then

$$\frac{s-a}{a}+\frac{s-b}{b}+\frac{s-c}{c}+\frac{s-d}{d}=\left\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right\}s-4.$$

2. If $s=a+b+c+d+\ldots\ldots$ to $n$ terms, then

$$\frac{s-a}{s}+\frac{s-b}{s}+\ldots\ldots=n-1.$$

3. If $a^2=y+z$, $b^2=z+x$, $c^2=x+y$, and $2s=a+b+c$, then

$$s(s-a)(s-b)(s-c)=4(xy+yz+zx).$$

4. If $2a=y+z$, $2b=z+x$, $2c=x+y$, then

$$a^4+b^4+c^4-2b^2c^2-2c^2a^2-2a^2b^2=-(x+y+z)xyz;\text{ and}$$
$$(x+y+z)(xy+yz+zx)-xyz=8abc.$$

5. If $x^2-yz=a^2$, $y^2-zx=b^2$, $z^2-xy=c^2$, then

$$\frac{a^2x+b^2y+c^2z}{x+y+z}=x^2+y^2+z^2-xy-yz-zx=a^2+b^2+c^2.$$

6. If $x=\frac{b-c}{a}$, $y=\frac{c-a}{b}$, $z=\frac{a-b}{c}$; then $xyz+x+y+z=0$.

7. If $y+z+u=ax$, $z+u+x=by$, $u+x+y=cz$, $x+y+z=du$,

$$\text{then }\frac{1}{a+1}+\frac{1}{b+1}+\frac{1}{c+1}+\frac{1}{d+1}=1.$$

8. If $a=\frac{x-y}{x+y}$, $b=\frac{y-z}{y+z}$, $c=\frac{z-x}{z+x}$, then

$$\frac{1+a}{1-1}\cdot\frac{1+b}{1-b}\cdot\frac{1+c}{1-c}=1.$$

9. If $2s=a+b+c$, then

$$(s-a)^2+(s-b)^2+(s-c)^2+s^2=a^2+b^2+c^2.$$

10. If $2s=a+b+c$, then

$$2(s-a)(s-b)+2(s-b)(s-c)+2(s-c)(s-a)=2s^2-a^2-b^2-c^2.$$

11. If $\frac{a}{x}(b-c)+\frac{b}{y}(c-a)+\frac{c}{z}(a-b)=0$; shew that

$$\frac{x}{a}(z-y)+\frac{y}{b}(x-z)+\frac{z}{c}(y-x)=0.$$

12. If $2s=a+b+c$, then shall

$$\frac{1}{s-a}+\frac{1}{s-b}+\frac{1}{s-c}+\frac{1}{s}=\frac{abc}{s(s-a)(s-b)(s-c)}.$$

13. If $\frac{b^2+c^2-a^2}{2bc}+\frac{c^2+a^2-b^2}{2ac}+\frac{a^2+b^2-c^2}{2ab}=1$,

then shall

$$(a+b-c)(b+c-a)(c+a-b)=0.$$

14. If $s=a+b+c$, shew that

$$s(s-2b)(s-2c)+s(s-2c)(s-2a)+s(s-2a)(s-2b)$$
$$-(s-2a)(s-2b)(s-2c)=8abc.$$

15. If $x+y=p$, and $xy=q$, then

$$x^2+y^2=p^2-2q,$$
$$x^3+y^3=p^3-3pq,$$
$$x^4+y^4=p^4-4p^2q+2q^2$$
$$x^5+y^5=p^5-5p^3q+pq^2:$$

16. If $a^2+b^2=c^2$, then shall

$$(a+b+c)(a+b-c)(a+c-b)(b+c-a)=4a^2b^2.$$

17. If $\frac{a^2+bc}{a^2-bc}+\frac{b^2+ca}{b^2-ca}+\frac{c^2+ab}{c^2-ab}=1$, then shall

$$\frac{a^2}{b^2+c^2-a^2}+\frac{b^2}{a^2+c^2-b^2}+\frac{c^2}{a^2+b^2-c}=-\frac{3}{2}.$$

18. If $(a^2-bc)(b^2-ca)(c^2-ab)=0$, then shall

$$\frac{1}{a^3}+\frac{1}{b^3}+\frac{1}{c^3}=\frac{a^3+b^3+c^3}{a^2b^2c^2}.$$

19. If $xyz=1$, then

$$(1+x+y^{-1})^{-1}+(1+y+z^{-1})^{-1}+(1+z+x^{-1})^{-1}=1.$$

20. If $a=(b+c)x$, $b=(c+a)y$, $c=(a+b)z$, then

$$1-xy-yz-zx-2xyz=0.$$

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We delayed the solution of 6, (2), of the October number that we might give some kindred problems with their solutions. We first give the problem referred to.

I. Let $S=\frac{1}{a}-\frac{1}{ab}+\frac{1}{abc}-\ldots\ldots$

$$=\frac{1}{a}-\frac{1}{a}\left[\frac{1}{b}-\frac{1}{b}\left\{\frac{1}{c}-\frac{1}{c}\left(\frac{1}{d}-\ldots\ldots\right.\right.\right.$$

$$=\frac{1}{a}-\frac{1}{a}\cdot k,\text{ say.}$$

$$=\frac{1}{\frac{a}{1-k}}$$

$$=\frac{1}{a+\frac{a}{\frac{1}{k}-1}}$$

Where $k=\frac{1}{b}-\frac{1}{b}\cdot l$, say

$$=\frac{1}{b+\frac{b}{\frac{1}{l}-1}}$$

$$\therefore\frac{1}{k}=b+\frac{b}{\frac{1}{l}-1}$$

$$\text{whence }S=\cfrac{1}{a+\cfrac{a}{b-1+\cfrac{b}{c-1+\ldots\ldots}}}$$

$$\text{Now }e^{-1}=\frac{1}{2}-\frac{1}{2.3}+\frac{1}{2.3.4}-\ldots$$

$$\therefore e^{-1}=\cfrac{1}{2+\cfrac{2}{2+\cfrac{3}{3+\ldots\ldots}}}$$

$$\text{or }e=2+\frac{2}{2+}\,\frac{3}{3+}\ldots$$

Again,

II. If $S=\frac{1}{a}+\frac{1}{ab}+\frac{1}{abc}+\ldots\ldots=\frac{1}{a}+\frac{1}{a}.k$, say

$$=\frac{1}{\frac{a}{1+k}}=\frac{1}{a-\frac{ak}{1+k}}$$

$$=\frac{1}{a-\frac{a}{1+\frac{1}{k}}}.$$

Similarly $\frac{1}{k}=b-\frac{b}{1+\frac{1}{l}}$, &c.