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Strength of Various Long Columns

Formulae Derived from the Results of a Large Number of Tests on Columns for Mild Steel and Wrought Iron—Value of the Constant in Euler's Formula for Different Conditions of the Bases—Elastic Limit of Normal Steel In Tension and Compression Similar

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THE accompanying article regarding the development of formula covering strength of long columns will be of interest to many readers.

Safe load	Equals	P
Area of cross-section	"	A
$\frac{P}{A}$	"	p
Maximum eccentricity or deflection	"	a
Width	"	h
Radius of gyration	"	r
Stress on outside fibre	"	f
Average stress on outside fibre	"	f′
Stress and outside fibre due to bending	"	f″
Length	"	L
Constant	"	K
Modulas of elasticity	"	E
Distortion of fibres or shrinkage under load	"	d
Angle	"	i

Radius=R

Referring to small sketch of column

$$i \text{ Equals } \frac{L}{R} \text{ Equals } \frac{d}{h} \text{ and R Equals } \frac{Lh}{d}$$

$$a \text{ Equals } 2R \sin^2 \frac{i}{4} \text{ Equals } 2R \text{ arc}^2 \frac{i}{4}$$ angles under consideration are small

$$\text{then a Equals } \frac{2Lh \times d \times d}{d \times 4h \times 4h} \text{ Equals } \frac{Ld}{8h} \quad \ldots\ldots\ldots \quad 1.$$

If the load be applied so that it is evenly distributed over the end of the column and in such a manner as to allow the column to move freely one way, and if it be assumed that the curve of a bent column can be figured as a parabola, as curves under consideration are very flat, then f′ on the inner side of the column will be

$\frac{2}{3}$f″ plus p Equals f′

but $\frac{2}{3}$f″ plus $\frac{2}{3}$p Equals $\frac{2}{3}$f

then $\frac{2}{3}$f″ plus $\frac{1}{3}$p Equals f′

$$\text{Then d Equals } \frac{(\frac{2}{3}f \text{ plus } \frac{1}{3}p)L}{E} \quad \ldots \text{ Inner side} \ldots\ldots \quad 2.$$

In the same way f′ on the outer side of the column will be

f′ Equals—$\frac{2}{3}$f plus $\frac{5}{3}$p

$$\text{Then d Equals } \frac{(-\frac{2}{3}f \text{ plus } \frac{5}{3}p)L}{E} \quad \ldots \text{ Outer side} \ldots\ldots \quad 3.$$

Subtracting equation No. 3 from No. 2 will give the difference in shrinkage between the inner and outer edges.

$$\text{Then d Equals } \frac{\frac{4}{3}(f-p)L}{E} \quad \ldots\ldots\ldots \quad 4.$$

substituting this value of d in equation No. 1

$$\text{then a Equals } \frac{(f-p)L^2}{6Eh} \quad \ldots\ldots\ldots \quad 51.$$

Formula for eccentric loading is

$$\text{p Equals } \frac{f}{1 \text{ plus } \frac{ah}{2r^2}} \quad \ldots\ldots\ldots \quad 6.$$

Substituting the value of in equation 5,

$$\text{Equation No. 6 becomes p Equals } \frac{12E}{\frac{(L)^2}{(r)}} \quad \ldots\ldots\ldots \quad 7.$$

which is the same form as Euler's formula, it being for columns with round ends

$$\text{p Equals } \frac{10E}{\frac{(L)^2}{(r)}} \quad \ldots\ldots\ldots \quad 8.$$

It is a question whether or not the constant 12 is in error owing to the asumptions made, although a column loaded as above mentioned would be stronger than a column with round ends.

Theoretically, for a perfect colunm, centrally loaded, the strength is constant for increasing lengths, this strength being "for practical purposes the true elastic limit of the material" until the critical length is reached under which the column bends indefinitely under its maximum load, when for any further increase in length the load which will produce this bending regularly diminishes in accordance with the law of Euler's curve.

The theoretical locus, therefore, of the maximum strength, for the value of P plotted to $\frac{L}{r}$ would be a horizontal line at the true elastic limit of the material, extended to an intersection with Euler's curve "plotted for the maximum value of E of the material and then down along this curve indefinitely.

A large number of tests of columns have been made and all possible care has been taken to insure that they are centrally loaded, but the result of these tests of short columns do not follow the above mentioned locus, as it would appear that the columns must have different values of E on each side which could be caused by minor defects. As soon as this is assumed this neutral axis is moved and the radius of gyration is decreased; a theoretical formula could be developed so that the minimum strength of a column, with different values of E, could be found, but it would then be necessary to find from actual tests the correct value of E to use; the same result can be obtained by making E in equation 3 equal to kE and formula 7 then becomes

$$\frac{L}{r} \text{Equals } \sqrt{\frac{12E\ (f-p)}{p\ (f-kp)}} \quad \ldots\ldots\ldots \quad 9.$$

This formula would then give the minimum strength of a centrally loaded column with different value of E.

The next step is to find the value of the constant in Euler's formula No. 8 for the different conditions of the