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Direct Design of Curvature of Arches

A Short and Accurate Analytical Method of Finding the Ordinates of the Curve for Concrete and Masonry Arches, With An Example

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IT has long been known that the Transformed Catenary is the curve of equilibrium for a linear arch under a vertical load-area of uniform weight per unit area. This may be shown as follows:—

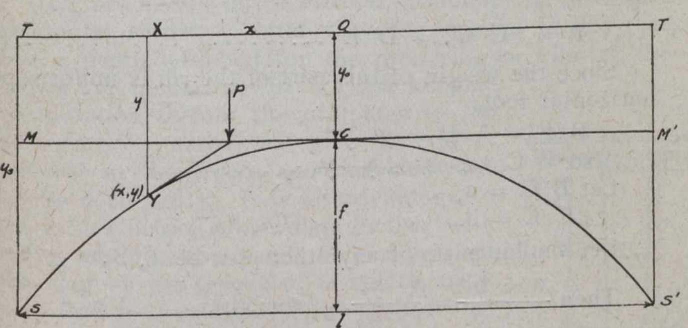

Fig. 1—Curve of Equilibrium for Homogeneous Loading

Let the linear arch S C S′ be in equilibrium under the load-area above it T T′ S′ C S.

Take O T axis of x.

O C axis of y.

Let A = area O C X Y

ω = weight per unit area of load-area.

H = constant horizontal thrust.

P = weight of load under O X.

Take a, so that $H = \omega a^2$ ....I.

Then $P = \omega A$.

$$\text{and } \frac{A}{a^2} = \frac{P}{H} = \frac{dy}{dx} \qquad \text{.........................II.}$$

$$\text{and } A = \int_{x = ox}^{x = o} y\, dx \qquad \text{.........................III.}$$

$$\therefore \frac{d^2 A}{dx^2} = \frac{dy}{dx} = \frac{A}{a^2}$$

Integrating

$$A = V e^{\frac{x}{a}} - W e^{-\frac{x}{a}}$$

Where V and W are constants to be determined:

When $x = o$, $A = o$, and $e^{\frac{x}{a}} = e^{-\frac{x}{a}} = 1$.

and the above equation can be put in the form

$$A = V\left(e^{\frac{x}{a}} - e^{-\frac{x}{a}}\right),$$

$$\therefore \frac{dA}{dx} = \frac{V}{a}\left(e^{\frac{x}{a}} + e^{-\frac{x}{a}}\right) = y \text{ from III.}$$

When $x = o$, $y = y_o$.

$$\therefore y_o = \frac{2V}{a}$$

and the above equation becomes:

$$\frac{y}{y_o} = \frac{e^{\frac{x}{a}} + e^{-\frac{x}{a}}}{2} \qquad \text{..........................IV.}$$

which is the equation to the curve.

This curve was made the basis of a direct method of design for masonry arches, where no tension in any part of the arch ring was desired, by Alexander and Thomson* whereby the line of resultant pressure for dead and live loads may be kept within the middle third of the ring. Otherwise small mention has been made of it in late works on arches.

In important arches it has often been considered worth while to fit the line of resultant pressure for the dead load by graphical methods to coincide with the middle line of the arch ring, so that under the dead load there shall be no bending moment at any section of the arch ring. This curve is chosen in order to be economical of material, for the principal stresses from other than the dead load are reversible moment stresses of about equal amounts, positive and negative. It is not, however, the arch of exactly minimum material where the dead loads alone are taken for the equilibrium curve, as there are exceptions to the above statement as to reversible stresses of equal amounts which may be taken account of. But any part of the distributed live load may be taken along with the dead load in forming the equilibrium curve just as easily as the dead load alone. It is therefore unnecessary for the purpose of this article to discuss further the arch of minimum material.

The curve of the middle line of the arch may be found so that it coincides with the equilibrium curve by the following analytical method. This applies to open spandrel arches where the weight of the floor system (as distinct from the spandrel vertical supports and the arch ring) is uniform per lineal foot and to earth-filled arches if the pressure of the filling is assumed to be vertical.

In these cases part of the dead weight is uniform per horizontal foot, the weight of the floor system and part of the arch ring, and that part of the spandrel supports

*"On Two-nosed Catenaries and their Application to the Design of Segmental Arches." Transactions of the Royal Irish Academy, Vol. XXIX., part III., 1888.